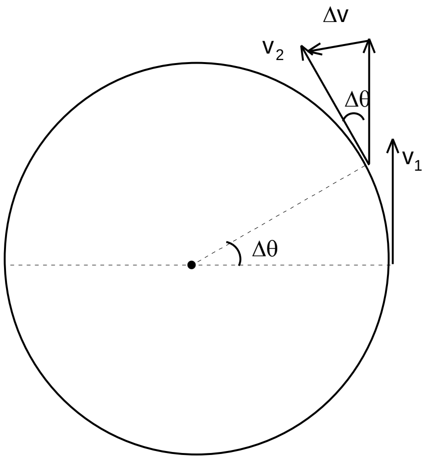
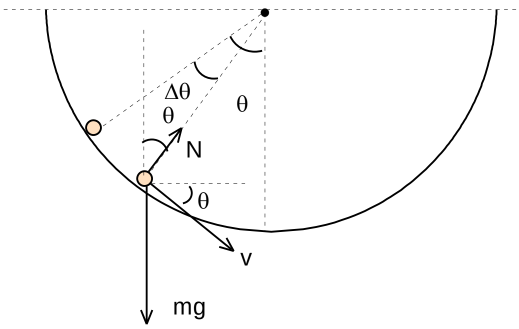


$$\frac{\Delta v}{\Delta t}=\frac{v\Delta\theta}{\Delta t}=v\frac{\Delta\theta}{\Delta t}=v\frac{\frac{v\Delta t}{r}}{\Delta t}=\frac{v^2}{r}$$



$$mg-N\cos\theta=m\frac{\Delta v_y}{\Delta t}\Rightarrow v_y=v\sin\theta$$

$$\Delta v_y=(v+\Delta v)\sin(\theta-\Delta\theta)-v\sin\theta=(v+\Delta v)(\sin\theta\cos\Delta\theta-\cos\theta\sin\Delta\theta)-v\sin\theta\approx(v+\Delta v)(\sin\theta-\cos\theta.\Delta\theta)-v\sin\theta=$$

$$\approx v\sin\theta+\Delta v\sin\theta-v\cos\theta.\Delta\theta-\Delta v\cos\theta.\Delta\theta-v\sin\theta\approx\Delta v\sin\theta-v\cos\theta.\Delta\theta$$

$$N\sin\theta=m\frac{\Delta v_x}{\Delta t}\Rightarrow v_x=v\cos\theta$$

$$\Delta v_x=(v+\Delta v)\cos(\theta-\Delta\theta)-v\cos\theta=(v+\Delta v)(\cos\theta\cos\Delta\theta+\sin\theta\sin\Delta\theta)-v\cos\theta\approx(v+\Delta v)(\cos\theta+\sin\theta.\Delta\theta)-v\cos\theta=$$

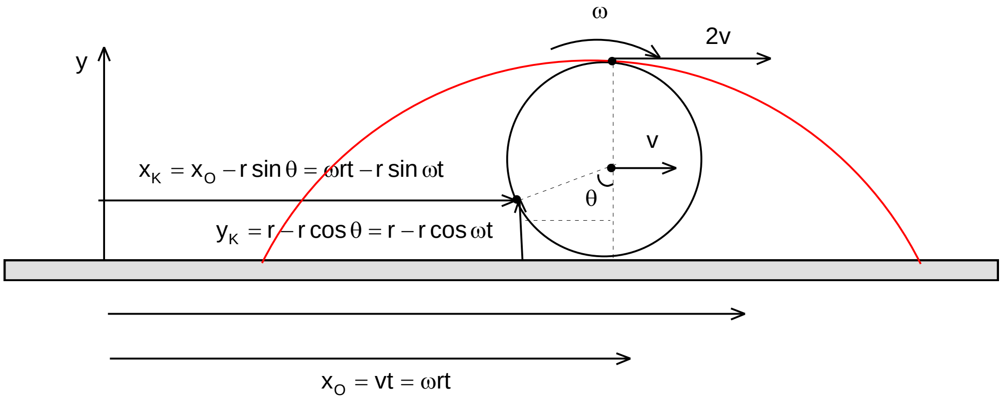
$$\approx v\cos\theta+\Delta v\cos\theta+v\sin\theta.\Delta\theta+\Delta v\sin\theta.\Delta\theta-v\cos\theta\approx\Delta v\cos\theta+v\sin\theta.\Delta\theta$$

$$mg-N\cos\theta=m\frac{(\Delta v\sin\theta-v\cos\theta.\Delta\theta)}{\Delta t}\Rightarrow mg\cos\theta-N\cos^2\theta=m\frac{(\Delta v\sin\theta\cos\theta-v\cos^2\theta.\Delta\theta)}{\Delta t}$$

$$N\sin\theta=m\frac{(\Delta v\cos\theta+v\sin\theta.\Delta\theta)}{\Delta t}\Rightarrow N\sin^2\theta=m\frac{(\Delta v\sin\theta\cos\theta+v\sin^2\theta.\Delta\theta)}{\Delta t}$$

$$N\sin^2\theta-(mg\cos\theta-N\cos^2\theta)=m\frac{(\Delta v\sin\theta\cos\theta+v\sin^2\theta.\Delta\theta)}{\Delta t}-m\frac{(\Delta v\sin\theta\cos\theta-v\cos^2\theta.\Delta\theta)}{\Delta t}$$

$$N-mg\cos\theta=m\frac{(v\sin^2\theta.\Delta\theta+v\cos^2\theta.\Delta\theta)}{\Delta t}=m\frac{v\Delta\theta}{\Delta t}=\frac{mv^2}{r}$$



$$x_K = x_O - r\sin\theta = \omega rt - r\sin\omega t \Rightarrow v_x = \omega r - \omega r\cos\omega t = \omega r(1-\cos\omega t) \Rightarrow \cos 2\alpha = 1-2\sin^2\alpha \Rightarrow \sin^2\alpha = \frac{1-\cos 2\alpha}{2}$$

$$y_K = r - r\cos\theta = r - r\cos\omega t \Rightarrow v_y = \omega r\sin\omega t$$

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{\left[\omega r(1-\cos\omega t)\right]^2 + (\omega r\sin\omega t)^2} = \omega r\sqrt{1-2\cos\omega t + \cos^2\omega t + \sin^2\omega t} = \omega r\sqrt{1-2\cos\omega t + 1} =$$

$$= \omega r\sqrt{2}\sqrt{1-\cos\omega t} = \omega r\sqrt{2}\sqrt{2\sin^2\frac{\omega t}{2}} = 2\omega r\sin\frac{\omega t}{2}$$

$$\ell = \int_0^T v\,dt = \int_0^T 2\omega r\sin\frac{\omega t}{2}dt = 4r\int_0^T \sin\frac{\omega t}{2}d\left(\frac{\omega t}{2}\right) = -4r\cos\frac{\omega t}{2}\Big|_0^{\frac{2\pi}{\omega}} = -4r\left(\cos\frac{\omega}{2}\frac{2\pi}{\omega} - \cos 0\right) = -4r(-1-1) = 8r$$

$$\frac{v^2}{r} = \frac{(2v)^2}{R} \Rightarrow \frac{v^2}{r} = \frac{4v^2}{R};\ R = 4r$$

$$d\ell = \sqrt{(dx)^2 + (dy)^2} = dx\sqrt{1+\left(\frac{dy}{dx}\right)^2} \Rightarrow \sin\theta = \frac{\tan\theta}{\sqrt{1+\tan^2\theta}} \Rightarrow \cos\theta = \frac{1}{\sqrt{1+\tan^2\theta}}$$

$$\frac{dy}{dx} = \tan\theta \Rightarrow \frac{d^2y}{dx^2} = \frac{1}{\cos^2\theta}\frac{d\theta}{dx} = (1+\tan^2\theta)d\theta = \left[1+\left(\frac{dy}{dx}\right)^2\right]\frac{d\theta}{dx} \Rightarrow d\theta = \frac{\frac{d^2y}{dx^2}dx}{1+\left(\frac{dy}{dx}\right)^2}$$

$$r = \frac{d\ell}{d\theta} = \frac{dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}}{\frac{\frac{d^2y}{dx^2}dx}{1+\left(\frac{dy}{dx}\right)^2}} = \frac{\sqrt{\left[1+\left(\frac{dy}{dx}\right)^2\right]^3}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{\sqrt{\left[1+\left(\frac{\dot y}{\dot x}\right)^2\right]^3}}{\left|\frac{\ddot y\dot x - \dot y\ddot x}{\dot x^3}\right|} = \frac{\sqrt{\left[\frac{\dot x^2+\dot y^2}{\dot x^2}\right]^3}}{\left|\frac{\ddot y\dot x - \dot y\ddot x}{\dot x^3}\right|} = \frac{\frac{\sqrt{(\dot x^2+\dot y^2)^3}}{\dot x^3}}{\left|\frac{\ddot y\dot x - \dot y\ddot x}{\dot x^3}\right|} = \frac{\sqrt{(\dot x^2+\dot y^2)^3}}{|\ddot y\dot x - \dot y\ddot x|}$$

$$x = x(t) \Rightarrow y = t(t)$$

$$\dot x = \frac{dx}{dt} \Rightarrow \dot y = \frac{dy}{dt} \Rightarrow \ddot x = \frac{d^2x}{dt^2} \Rightarrow \ddot y = \frac{d^2y}{dt^2}$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\dot y}{\dot x} \Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{\frac{d}{dt}\left(\frac{\dot y}{\dot x}\right)}{\frac{dx}{dt}} = \frac{\frac{\ddot y\dot x - \dot y\ddot x}{\dot x^2}}{\dot x} = \frac{\ddot y\dot x - \dot y\ddot x}{\dot x^3}$$

$$x_K = \omega rt - r\sin\omega t \Rightarrow y_K = r - r\cos\theta = r - r\cos\omega t$$

$$\dot x = v_x = \omega r(1-\cos\omega t) \Rightarrow \dot y = v_y = \omega r\sin\omega t$$

$$\ddot x = \omega^2 r\sin\omega t \Rightarrow \ddot y = \omega^2 r\cos\omega t$$

$$R = \frac{\sqrt{(\dot x^2+\dot y^2)^3}}{|\ddot y\dot x - \dot y\ddot x|} = \frac{\sqrt{\left\{\left[\omega r(1-\cos\omega t)\right]^2 + (\omega r\sin\omega t)^2\right\}^3}}{\left|\omega^2 r\cos\omega t.\omega r(1-\cos\omega t) - \omega r\sin\omega t.\omega^2 r\sin\omega t\right|} = \frac{\omega^3 r^3\sqrt{(1-2\cos\omega t + \cos^2\omega t + \sin^2\omega t)^3}}{\omega^3 r^2\left|\cos\omega t - \cos^2\omega t - \sin^2\omega t\right|} =$$

$$\frac{r\sqrt{(1-2\cos\omega t+1)^3}}{|\cos\omega t - 1|} = \frac{r\sqrt{\left[2(1-\cos\omega t)\right]^3}}{|1-\cos\omega t|} = \frac{r\sqrt{\left[2(1+1)\right]^3}}{|1+1|} = \frac{8r}{2} = 4r$$