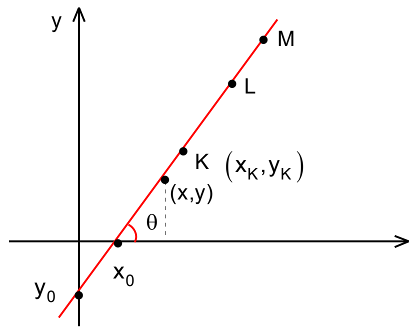
Kütle Merkezi



$$\frac{y-0}{x-x_0}=\frac{0-y_0}{0-x_0}=\frac{y_0}{x_0}\Rightarrow yx_0=-xy_0+x_0y_0\Rightarrow$$

$$yx_0+xy_0=x_0y_0\Rightarrow$$

$$\frac{yx_0}{x_0y_0}+\frac{xy_0}{x_0y_0}=\frac{x_0y_0}{x_0y_0}\Rightarrow\frac{x}{x_0}+\frac{y}{y_0}=1$$

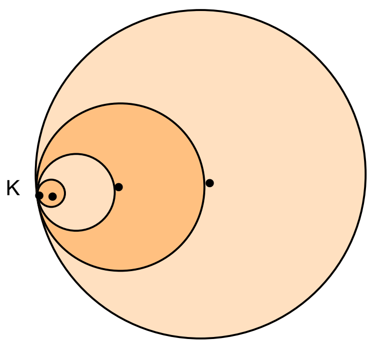
$$m_1\ell_1=m_2\ell_2\Rightarrow\frac{m_1}{m_2}=\frac{\ell_2}{\ell_1}$$

$$|KL|=\ell_1=\sqrt{(x_L-x_K)^2+(y_L-y_K)^2}\Rightarrow|LM|=\ell_2=\sqrt{(x_M-x_L)^2+(y_M-y_L)^2}$$

$$\frac{|KL|}{|LM|}=\frac{\ell_1}{\ell_2}=\lambda\Rightarrow\lambda=\frac{x_L-x_K}{x_M-x_L}\Rightarrow\lambda x_M-\lambda x_L=x_L-x_K\Rightarrow x_K+\lambda x_M=(1+\lambda)x_L\Rightarrow x_L=\frac{x_K+\lambda x_M}{1+\lambda}$$

$$x_L=\frac{x_K+\lambda x_M}{1+\lambda}=x_L=\frac{x_K+\frac{\ell_1}{\ell_2}x_M}{1+\frac{\ell_1}{\ell_2}}=\frac{x_K+\frac{m_2}{m_1}x_M}{1+\frac{m_2}{m_1}}=\frac{m_1x_K+m_2x_M}{m_1+m_2}=\frac{m_1x_1+m_2x_2}{m_1+m_2}$$

$$x_{km}=\frac{m_1x_1+m_2x_2+m_3x_3+....+m_nx_n}{m_1+m_2+m_3+...+m_n}$$

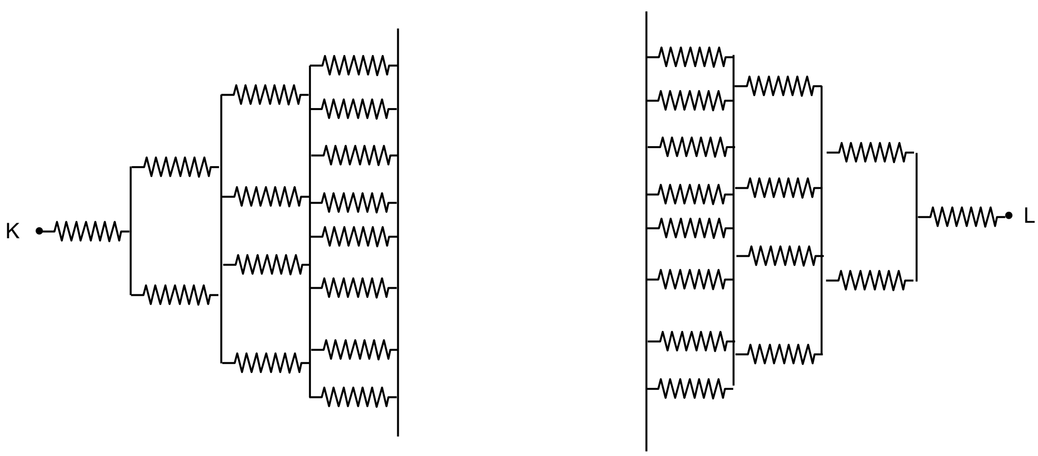


$m=\rho V=\rho\pi r^2 h$ r

$m_2=\rho V=\rho\pi\left(\frac{r}{2}\right)^2 h=\frac{m}{4}$ $\frac{r}{2}$

$m_2=\rho V=\rho\pi\left(\frac{r}{4}\right)^2 h=\frac{m}{16}$ $\frac{r}{4}$

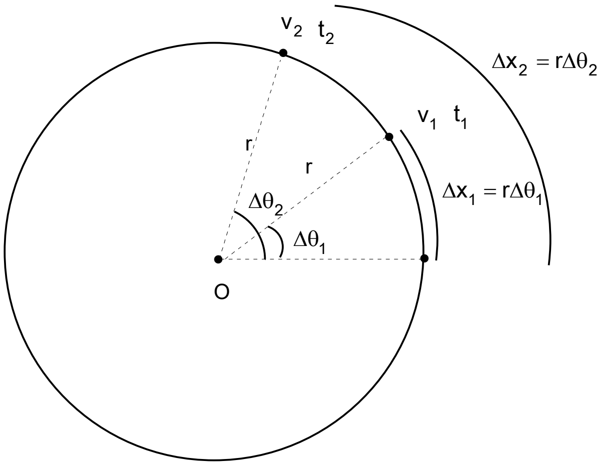
$$x_{km}=\frac{m.r+\frac{m}{4}.\frac{r}{2}+\frac{m}{16}.\frac{r}{4}+\frac{m}{64}.\frac{r}{8}+....}{m+\frac{m}{4}+\frac{m}{16}+\frac{m}{64}+...+}=\frac{mr\left(1+\frac{1}{8}+\frac{1}{64}+\frac{1}{512}+....\right)}{m\left(1+\frac{1}{4}+\frac{1}{16}+\frac{1}{64}+...\right)}=\frac{r\frac{1}{1-\frac{1}{8}}}{\frac{1}{1-\frac{1}{4}}}=\frac{\frac{8r}{7}}{\frac{4}{3}}=\frac{8r}{7}.\frac{3}{4}=\frac{6r}{7}$$

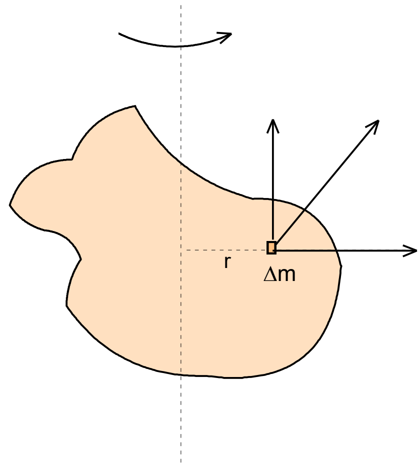


$r;\ \frac{r}{2};\ \frac{r}{4};\ \frac{r}{8}$ +++......++ $\frac{r}{8}+\frac{r}{4}+\frac{r}{2}+r$

$$N \gg 1 \Rightarrow \sum_{k=1}^{N} k \approx \frac{N^2}{2} \Rightarrow \sum_{k=1}^{N} k^2 \approx \frac{N^3}{3} \Rightarrow \sum_{k=1}^{N} k^3 \approx \frac{N^4}{4} \Rightarrow \sum_{k=1}^{N} k^4 \approx \frac{N^5}{5}$$

moment denklemi ve eylemsizlik momenti





$$\ell = 2\pi r \Rightarrow x = \theta r$$

$$v = \frac{\Delta x}{\Delta t} \Rightarrow a = \frac{\Delta v}{\Delta t} \Rightarrow F = ma$$

$$\omega = \frac{\Delta\theta}{\Delta t} \Rightarrow \alpha = \frac{\Delta\omega}{\Delta t} \Rightarrow M = I\alpha$$

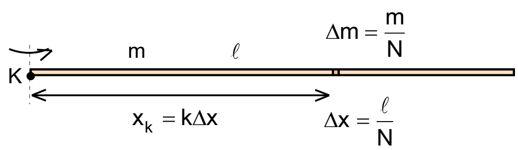
$$v = \frac{\Delta x_2 - \Delta x_1}{t_2 - t_1} = \frac{r\Delta\theta_2 - r\Delta\theta_1}{\Delta t} = \frac{r\left(\Delta\theta_2 - \Delta\theta_1\right)}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = \omega r$$

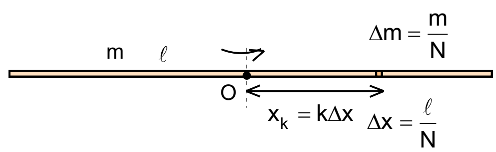
$$a = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\omega_2 r - \omega_1 r}{\Delta t} = \frac{r\left(\omega_2 - \omega_1\right)}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = \alpha r$$

$$\Delta M = Fr = \Delta m a r = \Delta m r^2 \alpha$$

$$M = \sum \Delta M = \sum \Delta m r^2 \alpha = \left(\sum \Delta m r^2\right)\alpha = J\alpha \Rightarrow J = \sum \Delta m r^2$$

Homojen bir çubuğu kütle merkezi ve eylemsizlik momenti





$$\Delta x = \frac{\ell}{N} \Rightarrow \Delta m = \frac{m}{N}$$

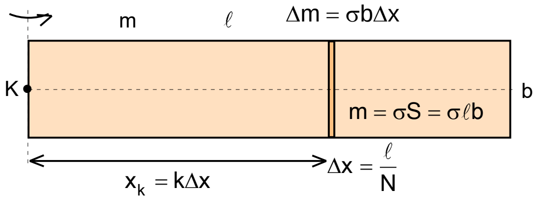
$$x_{km} = \frac{\Delta m.\Delta x + \Delta m.2\Delta x + \Delta m.3\Delta x + \Delta m.4\Delta x + ... + \Delta m.N\Delta x}{\Delta m + \Delta m + \Delta m + \Delta m + ... + \Delta m} = \frac{\Delta m\Delta x\left(1 + 2 + 3 + 4 + ... + N\right)}{m} = \frac{\frac{m}{N}.\frac{\ell}{N}.\frac{N^2}{2}}{m} = \frac{\ell}{2}$$

$$x_{km} = \frac{\int_0^{\ell} x\,dm}{\int_0^{\ell} \frac{m}{\ell}dx} = \frac{\int_0^{\ell} x.\frac{m}{\ell}dx}{\int_0^{\ell} \frac{m}{\ell}dx} = \frac{\frac{m}{\ell}\int_0^{\ell} x\,dx}{m} = \frac{1}{\ell}\left.\frac{x^2}{2}\right|_0^{\ell} = \frac{\ell}{2}$$

$$J_K = \sum_{k=1}^{N} \Delta m x^2 = \Delta m\left(\Delta x\right)^2 + \Delta m\left(2\Delta x\right)^2 + ... + \Delta m\left(N\Delta x\right)^2 = \Delta m\left(\Delta x\right)^2\left(1^2 + 2^2 + ... + N^2\right) = \frac{m}{N}\frac{\ell^2}{N^2}\frac{N^3}{3} = \frac{m\ell^2}{3}$$

$$J_O = 2\sum_{k=1}^{\frac{N}{2}} \Delta m x^2 = 2\left[\Delta m\left(\Delta x\right)^2 + \Delta m\left(2\Delta x\right)^2 + ... + \Delta m\left(\frac{N\Delta x}{2}\right)^2\right] = 2\Delta m\left(\Delta x\right)^2\left[1^2 + 2^2 + ... + \left(\frac{N}{2}\right)^2\right] = 2\frac{m}{N}\frac{\ell^2}{N^2}\frac{N^3}{3} = \frac{m\ell^2}{12}$$

$$J = J_0 + m z^2$$

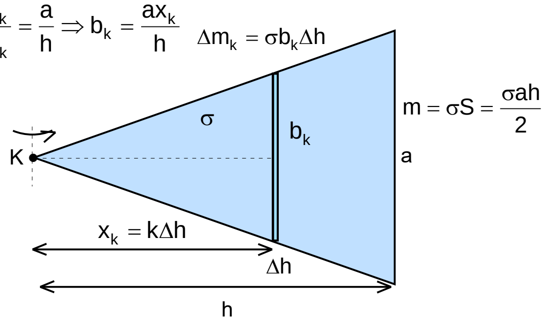


$$J_K=\sum_{k=1}^{N}\Delta m.\left(k\Delta x\right)^2+\sum_{k=1}^{N}\frac{1}{12}\Delta m.b^2=\sum_{k=1}^{N}\sigma b\Delta x.k^2\left(\Delta x\right)^2+\frac{1}{12}\sum_{k=1}^{N}\sigma b\Delta x.b^2=$$

$$=\sigma b\left(\Delta x\right)^3\sum_{k=1}^{N}k^2+\frac{\sigma b^3\Delta x}{12}\sum_{k=1}^{N}1=\sigma b\left(\Delta x\right)^3\frac{N^3}{3}+\frac{\sigma b^3\Delta x}{12}.N=\frac{\sigma b\ell^3}{3}+\frac{\sigma b^3\ell}{12}=\sigma b\ell\left(\frac{\ell^2}{3}+\frac{b^2}{12}\right)=m\left(\frac{\ell^2}{3}+\frac{b^2}{12}\right)$$

Aynı kalınlıktaki üçgenin kütle merkezi ve eylemsizlik momenti

$$\frac{b_k}{x_k}=\frac{a}{h}\Rightarrow b_k=\frac{ax_k}{h}\qquad \Delta m_k=\sigma b_k\Delta h$$



$$x_{km}=\frac{\sum_{k=1}^{N}\Delta m_k.k\Delta h}{\sum_{k=1}^{N}\Delta m_k}=\frac{\sum_{k=1}^{N}\sigma b_k\Delta h.k\Delta h}{\sum_{k=1}^{N}\sigma b_k\Delta h}=\frac{\sum_{k=1}^{N}\frac{ax_k}{h}\Delta h.k\Delta h}{\sum_{k=1}^{N}\frac{ax_k}{h}\Delta h}=\frac{\sum_{k=1}^{N}x_k\Delta h.k\Delta h}{\sum_{k=1}^{N}x_k\Delta h}=\frac{\sum_{k=1}^{N}k\Delta h.\Delta h.k\Delta h}{\sum_{k=1}^{N}k\Delta h.\Delta h}=\Delta h\frac{\sum_{k=1}^{N}k^2}{\sum_{k=1}^{N}k}=\Delta h\frac{\frac{N^3}{3}}{\frac{N^2}{2}}=\frac{2N\Delta h}{3}=\frac{2h}{3}$$

$$x_{km}=\frac{\int_0^h x.dm}{\int_0^h dm}=\frac{\int_0^h x.\sigma bdx}{\int_0^h \sigma bdx}=\frac{\int_0^h x\frac{ax}{h}dx}{\int_0^h \frac{ax}{h}dx}=\frac{\int_0^h x^2dx}{\int_0^h xdx}=\frac{\left.\frac{x^3}{3}\right|_0^h}{\left.\frac{x^2}{2}\right|_0^h}=\frac{\frac{h^3}{3}}{\frac{h^2}{2}}=\frac{2h}{3}$$

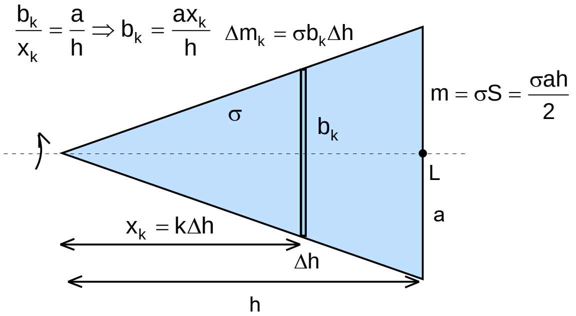
$$J_K=\sum_{k=1}^{N}\Delta m_k.\left(k\Delta h\right)^2+\sum_{k=1}^{N}\frac{1}{12}\Delta m_k.b_k{}^2=\sum_{k=1}^{N}\sigma b_k\Delta h.k^2\left(\Delta h\right)^2+\frac{1}{12}\sum_{k=1}^{N}\sigma b_k\Delta h.b_k{}^2=$$

$$=\sum_{k=1}^{N}\sigma.\frac{ax_k}{h}.k^2\left(\Delta h\right)^3+\frac{1}{12}\sum_{k=1}^{N}\sigma\left(\frac{ax_k}{h}\right)^3\Delta h=\frac{\sigma a}{h}\sum_{k=1}^{N}k\Delta h.k^2\left(\Delta h\right)^3+\frac{1}{12}\frac{\sigma a^3}{h^3}\sum_{k=1}^{N}\left(k\Delta h\right)^3.\Delta h=$$

$$=\frac{\sigma a\left(\Delta h\right)^4}{h}\sum_{k=1}^{N}k^3+\frac{1}{12}\frac{\sigma a^3\left(\Delta h\right)^4}{h^3}\sum_{k=1}^{N}k^3=\frac{\sigma a\left(\Delta h\right)^4}{h}\frac{N^4}{4}+\frac{1}{12}\frac{\sigma a^3\left(\Delta h\right)^4}{h^3}\frac{N^4}{4}=\frac{\sigma ah^4}{4h}+\frac{1}{48}\frac{\sigma a^3h^4}{h^3}=$$

$$=\frac{\sigma ah}{2}\frac{h^2}{2}+\frac{1}{24}\frac{\sigma ah}{2}.a^2=\frac{mh^2}{2}+\frac{ma^2}{24}=m\left(\frac{h^2}{2}+\frac{a^2}{24}\right)$$

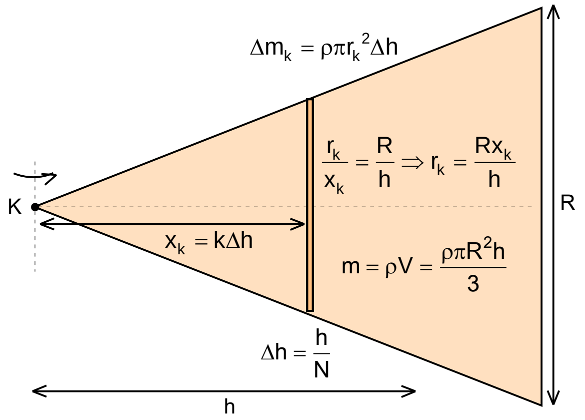
$$\frac{b_k}{x_k}=\frac{a}{h}\Rightarrow b_k=\frac{ax_k}{h}\qquad \Delta m_k=\sigma b_k\Delta h$$



$$J_L = \sum_{k=1}^{N}\frac{1}{12}\Delta m_k.b_k^2 = \frac{1}{12}\sum_{k=1}^{N}\sigma b_k \Delta h.b_k^2 = \frac{1}{12}\sum_{k=1}^{N}\sigma\left(\frac{ax_k}{h}\right)^3\Delta h = \frac{1}{12}\frac{\sigma a^3}{h^3}\sum_{k=1}^{N}(k\Delta h)^3.\Delta h =$$

$$= \frac{1}{12}\frac{\sigma a^3(\Delta h)^4}{h^3}\sum_{k=1}^{N}k^3 = \frac{1}{12}\frac{\sigma a^3(\Delta h)^4}{h^3}\frac{N^4}{4} = \frac{1}{48}\frac{\sigma a^3 h^4}{h^3} = \frac{1}{24}\frac{\sigma a h}{2}.a^2 = \frac{ma^2}{24}$$

Homojen koninin kütle merkezi ve eylemsizlik momenti



$$\rho = \frac{m}{V} = \frac{m}{\frac{\pi R^2 h}{3}} = \frac{3m}{\pi R^2 h} \Rightarrow \frac{r_k}{k\Delta h} = \frac{R}{h} \Rightarrow r_k = \frac{k\Delta h R}{h} \Rightarrow \Delta m_k = \rho\Delta V_k = \rho\pi r_k^2\Delta h = \frac{\rho\ell\left(k^2\Delta h\right)^3 R^2}{h^2}$$

$$x_{km} = \frac{\sum_{k=1}^{N}\Delta m_k.k\Delta h}{\sum_{k=1}^{N}\Delta m_k} = \frac{\sum_{k=1}^{N}\rho\pi r_k^2\Delta h.k\Delta h}{\sum_{k=1}^{N}\rho\pi r_k^2\Delta h} = \frac{\sum_{k=1}^{N}\left(\frac{Rx_k}{h}\right)^2\Delta h.k\Delta h}{\sum_{k=1}^{N}\left(\frac{Rx_k}{h}\right)^2\Delta h} = \frac{\sum_{k=1}^{N}(k\Delta h)^2\Delta h.k\Delta h}{\sum_{k=1}^{N}(k\Delta h)^2\Delta h} = \frac{\sum_{k=1}^{N}k^3(\Delta h)^4}{\sum_{k=1}^{N}k^2(\Delta h)^3} = \Delta h\frac{\sum_{k=1}^{N}k^3}{\sum_{k=1}^{N}k^2} = \Delta h\frac{\frac{N^4}{4}}{\frac{N^3}{3}} = \frac{3N\Delta h}{4} = \frac{3h}{4}$$

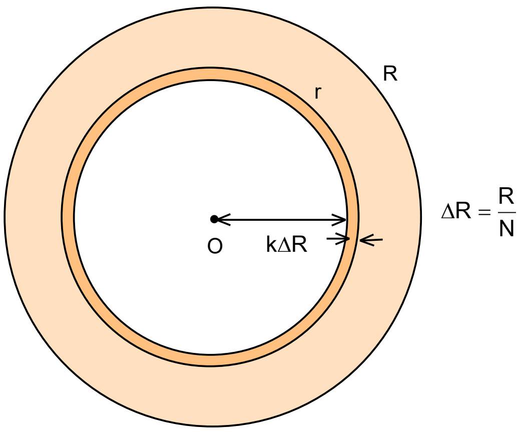
$$J_K = \sum_{k=1}^{N}\Delta m_k.(k\Delta h)^2 + \sum_{k=1}^{N}\frac{1}{12}\Delta m_k.(2r_k)^2 = \sum_{k=1}^{N}\rho\pi r_k^2\Delta h.(k\Delta h)^2 + \sum_{k=1}^{N}\frac{1}{12}\Delta m_k.4r_k^2 =$$

$$\sum_{k=1}^{N}\rho\pi\left(\frac{Rx_k}{h}\right)^2\Delta h.k^2(\Delta h)^2 + \frac{1}{12}\sum_{k=1}^{N}\rho\pi R^2\Delta h.4\left(\frac{Rx_k}{h}\right)^2 = \frac{\rho\pi R^2}{h^2}\sum_{k=1}^{N}(k\Delta h)^2\Delta h.k^2(\Delta h)^2 + \frac{\rho\pi R^4}{3h^2}\sum_{k=1}^{N}\Delta h.(k\Delta h)^2 =$$

$$= \frac{\rho\pi R^2(\Delta h)^5}{h^2}\sum_{k=1}^{N}k^4 + \frac{\rho\pi R^4(\Delta h)^3}{3h^2}\sum_{k=1}^{N}k^2 = \frac{\rho\pi R^2(\Delta h)^5}{h^2}\frac{N^4}{5} + \frac{\rho\pi R^4(\Delta h)^3}{3h^2}\frac{N^3}{3} = \frac{\rho\pi R^2 h^5}{5h^2} + \frac{\rho\pi R^4 h^3}{9h^2} =$$

$$= \frac{\rho\pi R^2 h}{3}\frac{3h^2}{5} + \frac{\rho\pi R^2 h}{3}\frac{R^2}{3} = m\frac{3h^2}{5} + m\frac{R^2}{3} = m\left(\frac{3h^2}{5} + \frac{R^2}{3}\right)$$

Kürenin hacmi



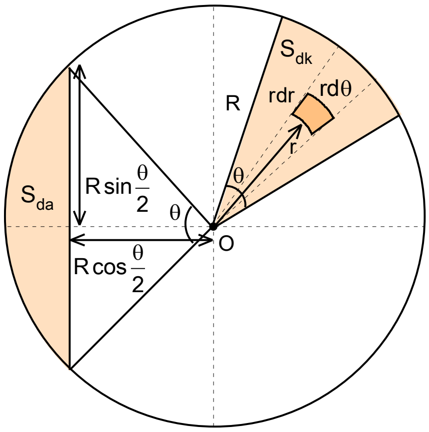
$$S = 4\pi r^2$$

$$\Delta V = 4\pi(k\Delta R)^2.\Delta R = 4\pi(\Delta R)^3.k^2$$

$$V = 4\pi(\Delta R)^3 + 4\pi(\Delta R)^3.2^2 + 4\pi(\Delta R)^3.3^2 + 4\pi(\Delta R)^3.4^2 + ... + 4\pi(\Delta R)^3.N^2 =$$

$$4\pi(\Delta R)^3\left(1 + 2^2 + 3^2 + 4^2 + ... + N^2\right) = 4\pi(\Delta R)^3\frac{N^3}{3} = \frac{4\pi(N\Delta R)^3}{3} = \frac{4\pi R^3}{3}$$

Dairesel kesit alanı, kısmi dairesel kesit alanı ve çemberin çevresi



$$S_{dk}=\int_0^R\int_0^\theta r\,dr\,d\theta=\frac{r^2}{2}\Big|_0^R.\theta\Big|_0^\theta=\frac{R^2\theta}{2}\ ;\ \theta=2\pi \text{ için};\ S=\pi R^2$$

$$S_{da}=S_{dk}-\frac{2R\sin\frac{\theta}{2}.R\cos\frac{\theta}{2}}{2}=\frac{R^2\theta}{2}-\frac{R^2\sin\theta}{2}=\frac{R^2(\theta-\sin\theta)}{2}$$

$x^2+y^2=R^2$; $y=\sqrt{R^2-x^2}$

$$d\ell=\sqrt{dx^2+dy^2}=dx\sqrt{1+\left(\frac{dy}{dx}\right)^2}\ ;\ \frac{dy}{dx}=-\frac{x}{\sqrt{R^2-x^2}}$$

$$\ell=\int_{-R}^{R}\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx\ 2=\int_{-R}^{R}\sqrt{1+\left(-\frac{x}{\sqrt{R^2-x^2}}\right)^2}\,dx=2\int_{-R}^{R}\frac{R\,dx}{\sqrt{R^2-x^2}}$$

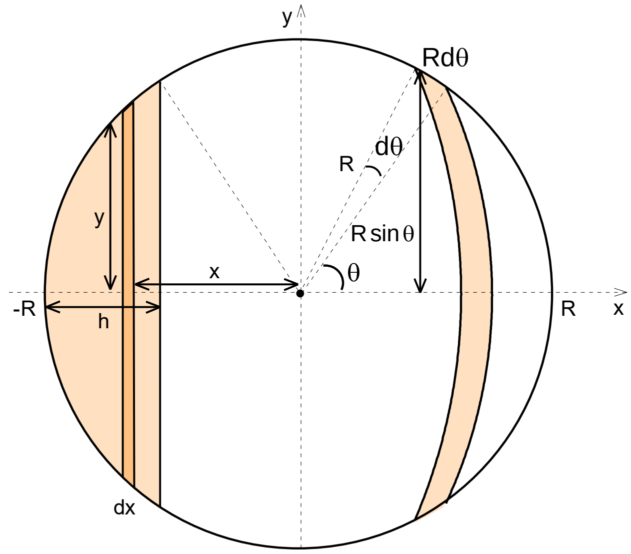
x=Rcosz; dx= -Rsinzdz

-R=Rcosz; z=π

R=Rcosz; z=0

$$\ell=-2\int_\pi^0\frac{R.R\sin z\,dz}{\sqrt{R^2-R^2\cos^2 z}}=-2R\int_\pi^0 dz=-2R\,z\Big|_\pi^0=-2R(0-\pi)=2\pi R$$

Katı açı, kürenin hacmi ve küreden bir kesitin hacmi



$$d\Omega=\frac{dS}{R^2}=\frac{2\pi R\sin\theta.Rd\theta}{R^2}=\frac{2\pi R^2\sin\theta d\theta}{R^2}=2\pi\sin\theta d\theta$$

$$\Omega=\int_0^\pi 2\pi\sin\theta d\theta=-2\pi\cos\theta\Big|_0^\pi=-2\pi[\cos\pi-\cos 0]=4\pi$$

$y=\sqrt{R^2-x^2}$

$$V=\int_{-R}^{R}\pi y^2dx=\int_{-R}^{R}\pi\left(R^2-x^2\right)dx=\pi R^2\,x\Big|_{-R}^{R}-\pi\frac{x^3}{3}\Big|_{-R}^{R}=$$

$$=\pi R^2(R+R)-\frac{\pi}{3}\left(R^3+R^3\right)=\frac{4\pi R^3}{3}$$

$$V_h=\int_{-R}^{-(R-h)}\pi\left(R^2-x^2\right)dx=\pi R^2x\Big|_{-R}^{-(R-h)}-\frac{\pi x^3}{3}\Big|_{-R}^{-(R-h)}=$$

$$=\pi h^2\left(R-\frac{h}{3}\right)$$

.

Bir rezistanstan geçen akım belli sürede sıfır değerinden maksimum değerine kadar düzgün olarak artıyor.

Bu sürenin ikinci yarısında açığa çıkan ısı, sürenin ilk yarısında açığa çıkan ısının kaç katıdır?

$$I_t=\frac{I_m t}{T}\Rightarrow P_t=I_t{}^2\Re=\frac{I_m{}^2t^2\Re}{T^2}\Rightarrow dQ=P_tdt=\frac{I_m{}^2\Re t^2dt}{T^2}\Rightarrow Q_m=\int_0^T\frac{I_m{}^2\Re t^2dt}{T^2}=\frac{I_m{}^2\Re t^3}{3T^2}\Big|_0^T=\frac{I_m{}^2\Re T}{3}$$

$$Q_1=\int_0^{\frac{T}{2}}\frac{I_m{}^2\Re t^2dt}{T^2}=\frac{I_m{}^2\Re t^3}{3T^2}\Big|_0^{\frac{T}{2}}=\frac{I_m{}^2\Re}{T^2}\frac{T^3}{24}=\frac{I_m{}^2\Re T}{24};\ Q_2=\frac{I_m{}^2\Re}{T^2}\int_{\frac{T}{2}}^{T}t^2dt=\frac{I_m{}^2\Re}{T^2}.\frac{t^3}{3}\Big|_{\frac{T}{2}}^{T}=\frac{I_m{}^2\Re}{T^2}\left(\frac{T^3}{3}-\frac{T^3}{24}\right)=\frac{7I_m{}^2\Re T}{24}$$

$$\frac{Q_2}{Q_1}=7$$

$$N=\frac{T}{\Delta t}\Rightarrow I_k=\frac{I_m k\Delta t}{T}\Rightarrow P_k=I_k{}^2\Re=\frac{I_m{}^2(k\Delta t)^2\Re}{T^2}\Rightarrow\Delta Q_k=P_k\Delta t=\frac{I_m{}^2\Re(\Delta t)^3k^2}{T^2}$$

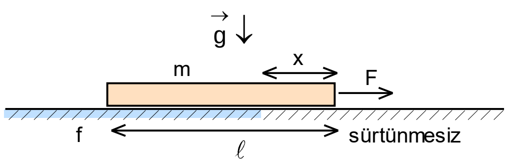
$$Q=\sum_{k=1}^{N}\Delta Q_k=\sum_{k=1}^{N}\frac{I_m{}^2\Re(\Delta t)^3k^2}{T^2}=\frac{I_m{}^2\Re(\Delta t)^3}{T^2}\sum_{k=1}^{N}k^2=\frac{I_m{}^2\Re(\Delta t)^3}{T^2}\frac{N^3}{3}=\frac{I_m{}^2\Re(\Delta t)^3}{T^2}\frac{1}{3}\left(\frac{T}{\Delta t}\right)^3=\frac{I_m{}^2\Re T}{3}$$

Bir ortamda v_0 ilk hızı verilen bir cisime etki eden direniş kuvveti F_d =kv dir.

Bu cismin zamana bağlı aldığı yol ve hızı nedir?

$$m\frac{dv}{dt} = -kv;\ m\int_{v_0}^{v}\frac{dv}{v} = -k\int_0^t dt\ ;\ m.\ln\frac{v}{v_0} = -kt;\ v={}_0\ e^{-\frac{kt}{m}}$$

$$x=\int_0^t vdt = \int_0^t v_0 e^{-\frac{kt}{m}}dt = \frac{mv_0}{k}\left(1-e^{-\frac{kt}{m}}\right)$$



$$F-\frac{fmg(\ell-x)}{\ell} = ma = m\frac{dv}{dt} = m\frac{dv}{dx}\frac{dx}{dt} = mv\frac{dv}{dx}$$

$$\int_0^v vdv = \int_0^x\left[\frac{F}{m}-\frac{fg(\ell-x)}{\ell}\right]dx = \left(\frac{Fx}{m}-\frac{fgx}{\ell}+\frac{fgx^2}{2\ell}\right)\Bigg|_0^x$$

$$\frac{v^2}{2} = \frac{Fx}{m}-\frac{fgx}{\ell}+\frac{fgx^2}{2\ell}\ ;\ v=2\sqrt{\left(\frac{F}{m}-\frac{fg}{\ell}\right)x+\frac{fgx^2}{2\ell}}$$

Viskozitesi η ve özkütlesi ρ_0 olan bir sıvı içinde düşey yönde, özkütlesi ρ olan küresel bir cisim, hareket v hızı ile doğru orantılı direniş $F_d = 6\pi\eta rv$ (Stoks kuvveti) kuvvetin etkisinin altında hareket etmektedir.

Cisim sıvı içinde serbest bırakıldıktan sonra zamana bağlı olan hız ve ulaştığı limit sabit hızı nedir?

$$m\frac{dv}{dt} = mg - 6\pi\eta rv - \rho_0 gV;\ m=\rho gV;\ V=\frac{4\pi r^3}{3}$$

$$\frac{4\pi r^3\rho}{3}\frac{dv}{dt} = \frac{4\pi r^3(\rho-\rho_0)g}{3} - 6\pi\eta rv;\ \frac{dv}{dt} = \frac{(\rho-\rho_0)g}{\rho} - \frac{9\eta v}{2\rho r^2}$$

$$\int_0^v \frac{dv_y}{\frac{(\rho-\rho_0)g}{\rho}-\frac{9\eta v}{2\rho r^2}} = \int_0^t dt\ ;\ -\frac{2\rho r^2}{9\eta}\int_0^v\frac{d\left[\frac{(\rho-\rho_0)g}{\rho}-\frac{9\eta v}{2\rho r^2}\right]}{\frac{(\rho-\rho_0)g}{\rho}-\frac{9\eta v}{2\rho r^2}} = t;\ \frac{2\rho r^2}{9\eta}\ln\left[\frac{(\rho-\rho_0)g}{\rho}-\frac{9\eta v}{2\rho r^2}\right]\Bigg|_0^v = -t$$

$$\ln\frac{\frac{(\rho-\rho_0)g}{\rho}-\frac{9\eta v}{2\rho r^2}}{\frac{(\rho-\rho_0)g}{\rho}} = -\frac{9\eta t}{2\rho r^2}\ ;\ v=\frac{2(\rho-\rho_0)gr^2}{9\eta}\left(1-e^{-\frac{9\eta t}{2\rho r^2}}\right) = v_0\left(1-e^{-\frac{t}{\tau}}\right);\ v_0=\frac{2(\rho-\rho_0)gr^2}{9\eta}\ ;\ \tau=\frac{2\rho gr^2}{9\eta}$$

$$x=\int_0^t vdt = \int_0^t v_0\left(1-e^{-\frac{t}{\tau}}\right)dt = v_0\left(t+\tau e^{-\frac{t}{\tau}}\right)\Bigg|_0^t = v_0\left(t+\tau e^{-\frac{t}{\tau}}-\tau\right)$$

Hava ortamında serbest olarak bırakılan m kütleli cisme etki eden direniş kuvveti $F_d = kv^2$ ile verilmektedir. Burada k bir sabittir.

Buna göre cismin hızı ve aldığı yol zamana göre nasıl değişir?

$$mg - F_d = ma;\; mg - kv^2 = m\frac{dv}{dt};\; dt = \frac{mdv}{mg-kv^2} = \frac{mdv}{mg\left(1-\frac{kv^2}{mg}\right)} = \frac{\sqrt{\frac{mg}{k}}\,d\left(v\sqrt{\frac{k}{mg}}\right)}{g\left[1-\left(v\sqrt{\frac{k}{mg}}\right)^2\right]} = \sqrt{\frac{m}{gk}}\frac{dz}{1-z^2}$$

$$t = \sqrt{\frac{m}{gk}}\int_0^z \frac{dz}{1-z^2} = \sqrt{\frac{m}{gk}}\int_0^z \frac{\left[(1+z)+(1-z)\right]dz}{2(1+z)(1-z)} = \frac{1}{2}\sqrt{\frac{m}{gk}}\left(\int_0^z \frac{dz}{1+z} + \int_0^z \frac{dz}{1-z}\right) = \frac{1}{2}\sqrt{\frac{m}{gk}}\left(\int_0^z \frac{d(1+z)}{1+z} - \int_0^z \frac{d(1-z)}{1-z}\right) =$$

$$= \frac{1}{2}\sqrt{\frac{m}{gk}}\left[\ln(1+z) - \ln(1-z)\right]\Big|_0^z = \frac{1}{2}\sqrt{\frac{m}{gk}}\ln\frac{1+z}{1-z} = \frac{1}{2}\sqrt{\frac{m}{gk}}\ln\frac{1+v\sqrt{\frac{k}{mg}}}{1-v\sqrt{\frac{k}{mg}}}$$

$$\frac{1+v\sqrt{\frac{k}{mg}}}{1-v\sqrt{\frac{k}{mg}}} = e^{2t\sqrt{\frac{gk}{m}}};\; 1+v\sqrt{\frac{k}{mg}} = \left(1-v\sqrt{\frac{k}{mg}}\right)e^{2t\sqrt{\frac{gk}{m}}};\; v\sqrt{\frac{k}{mg}}\left(1+e^{2\sqrt{\frac{gk}{m}}}\right) = e^{2t\sqrt{\frac{gk}{m}}} - 1;\; v = \sqrt{\frac{mg}{k}}\frac{e^{t\sqrt{\frac{gk}{m}}} - e^{-t\sqrt{\frac{gk}{m}}}}{e^{t\sqrt{\frac{gk}{m}}} + e^{-t\sqrt{\frac{gk}{m}}}}$$

$$v = \sqrt{\frac{mg}{k}}\,th\left(t\sqrt{\frac{gk}{m}}\right)$$

$$x = \int_0^t vdt = \int_0^t \sqrt{\frac{mg}{k}}\,th\left(t\sqrt{\frac{gk}{m}}\right)dt = \int_0^t \sqrt{\frac{mg}{k}}\frac{d\left[ch\left(t\sqrt{\frac{gk}{m}}\right)\right]}{ch\left(t\sqrt{\frac{gk}{m}}\right)} = \sqrt{\frac{mg}{k}}\ln\left[ch\left(t\sqrt{\frac{gk}{m}}\right)\right]$$

$$e^{ix} = \cos x + i\sin x \Rightarrow e^{-ix} = \cos x - i\sin x$$

$$\cos x = \frac{e^{ix}+e^{-ix}}{2} \Rightarrow \sin x = \frac{e^{ix}-e^{-ix}}{2i}$$

$$shx = \frac{e^x-e^{-x}}{2} \Rightarrow chx = \frac{e^x+e^{-x}}{2} \Rightarrow thx = \frac{shx}{chx} = \frac{e^x-e^{-x}}{e^x+e^{-x}} \Rightarrow cthx = \frac{chx}{shx} = \frac{e^x+e^{-x}}{e^x-e^{-x}}$$

$$ch^2x - sh^2x = \left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2 = \frac{e^{2x}+2e^xe^{-x}+e^{-2x}}{4} - \frac{e^{2x}-2e^xe^{-x}+e^{-2x}}{4} = 1$$

$$thx.cthx = \frac{e^x-e^{-x}}{e^x+e^{-x}}\frac{e^x+e^{-x}}{e^x-e^{-x}} = 1$$

$$(shx)' = \left(\frac{e^x-e^{-x}}{2}\right)' = \frac{e^x+e^{-x}}{2} = chx$$

$$(chx)' = \left(\frac{e^x+e^{-x}}{2}\right)' = \frac{e^x-e^{-x}}{2} = shx$$

$$(thx)' = \left(\frac{shx}{chx}\right)' = \frac{ch^2x - sh^2x}{ch^2x}' = \frac{1}{ch^2x}$$

$$(cthx)' = \left(\frac{chx}{shx}\right)' = \frac{sh^2x - ch^2x}{sh^2x}' = -\frac{1}{sh^2x}$$