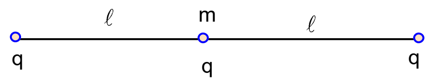


$$ma = -\left[\frac{q^2}{4\pi\varepsilon_0(\ell - x)^2} - \frac{q^2}{4\pi\varepsilon_0(\ell + x)^2}\right] = -\left[\frac{q^2}{4\pi\varepsilon_0\ell^2\left(1-\frac{x}{\ell}\right)^2} - \frac{q^2}{4\pi\varepsilon_0\ell^2\left(1+\frac{x}{\ell}\right)^2}\right] \approx$$

$$\approx -\left[\frac{q^2}{4\pi\varepsilon_0\ell^2\left(1-\frac{2x}{\ell}\right)} - \frac{q^2}{4\pi\varepsilon_0\ell^2\left(1+\frac{2x}{\ell}\right)}\right] \approx -\left[\frac{q^2}{4\pi\varepsilon_0\ell^2}\left(1+\frac{2x}{\ell}\right) - \frac{q^2}{4\pi\varepsilon_0\ell^2}\left(1-\frac{2x}{\ell}\right)\right] = -\frac{4q^2}{4\pi\varepsilon_0\ell^3}x$$

$$\ddot{x} + \frac{4q^2}{4\pi\varepsilon_0 m\ell^3}x = 0$$

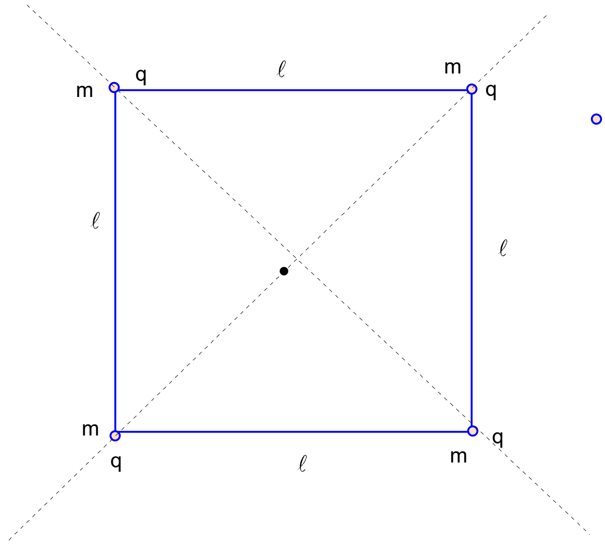
$$\mathcal{E} = \frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0(\ell - x)} + \frac{q^2}{4\pi\varepsilon_0(\ell + x)} + \frac{q^2}{4\pi\varepsilon_0.2\ell} = \frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\ell\left(1-\frac{x}{\ell}\right)} + \frac{q^2}{4\pi\varepsilon_0\ell\left(1+\frac{x}{\ell}\right)} + \frac{q^2}{4\pi\varepsilon_0.2\ell} \approx$$

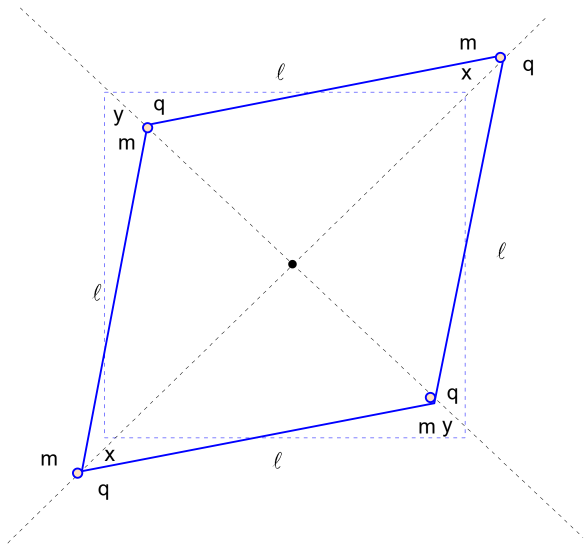
$$\approx \frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\ell}\left(1+\frac{x}{\ell}\right) + \frac{q^2}{4\pi\varepsilon_0\ell}\left(1-\frac{x}{\ell}\right) + \frac{q^2}{4\pi\varepsilon_0.2\ell} \approx$$

$$\mathcal{E} = \frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0(\ell - x)} + \frac{q^2}{4\pi\varepsilon_0(\ell + x)} + \frac{q^2}{4\pi\varepsilon_0.2\ell} = \frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0}\left[\frac{1}{\ell - x} + \frac{1}{\ell + x}\right] + \frac{q^2}{4\pi\varepsilon_0.2\ell} =$$

$$= \frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0}\left(\frac{\ell + x + \ell - x}{\ell^2 - x^2}\right) + \frac{q^2}{4\pi\varepsilon_0.2\ell} = \frac{mv^2}{2} + \frac{2\ell q^2}{4\pi\varepsilon_0}\frac{1}{\ell^2\left(1-\frac{x^2}{\ell^2}\right)} + \frac{q^2}{4\pi\varepsilon_0.2\ell} \approx \frac{mv^2}{2} + \frac{2\ell q^2}{4\pi\varepsilon_0}\frac{1}{\ell^2}\left(1+\frac{x^2}{\ell^2}\right) + \frac{q^2}{4\pi\varepsilon_0.2\ell} =$$

$$= \frac{mv^2}{2} + \frac{2q^2x^2}{4\pi\varepsilon_0\ell^3} + \frac{2q^2}{4\pi\varepsilon_0\ell} + \frac{q^2}{4\pi\varepsilon_0.2\ell} = \frac{mv^2}{2} + \frac{4q^2}{4\pi\varepsilon_0\ell^3}\frac{x^2}{2} + \frac{2q^2}{4\pi\varepsilon_0\ell} + \frac{q^2}{4\pi\varepsilon_0.2\ell} \Rightarrow \omega^2 = \frac{k}{m} = \frac{\frac{4q^2}{4\pi\varepsilon_0\ell^3}}{m}$$





$$\mathcal{E} = 2\frac{mv^2}{2} + 2\frac{mu^2}{2} + \frac{q^2}{4\pi\varepsilon_0\left(\ell\sqrt{2}+2x\right)} + \frac{q^2}{4\pi\varepsilon_0\left(\ell\sqrt{2}-2y\right)} + \frac{4q^2}{4\pi\varepsilon_0\ell} =$$

$$\ell^2 = \left(\frac{\ell\sqrt{2}}{2}+x\right)^2 + \left(\frac{\ell\sqrt{2}}{2}-y\right)^2 = \frac{\ell^2}{2} + \ell x\sqrt{2} + x^2 + \frac{\ell^2}{2} - \ell y\sqrt{2} + y^2 \Rightarrow 0 = \ell x\sqrt{2} + x^2 - \ell y\sqrt{2} + y^2 \Rightarrow 0 = \ell x\sqrt{2} - \ell y\sqrt{2} \Rightarrow x = y$$

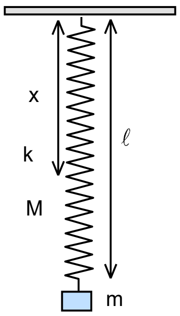
$$\mathcal{E} = 4\frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\left(\ell\sqrt{2}+2x\right)} + \frac{q^2}{4\pi\varepsilon_0\left(\ell\sqrt{2}-2x\right)} + \frac{4q^2}{4\pi\varepsilon_0\ell} = 4\frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\ell\sqrt{2}\left(1+\frac{2x}{\ell\sqrt{2}}\right)} + \frac{q^2}{4\pi\varepsilon_0\ell\sqrt{2}\left(1-\frac{2x}{\ell\sqrt{2}}\right)} + \frac{4q^2}{4\pi\varepsilon_0\ell} =$$

$$= 4\frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\ell\sqrt{2}\left(1+\frac{\sqrt{2}x}{\ell}\right)} + \frac{q^2}{4\pi\varepsilon_0\ell\sqrt{2}\left(1-\frac{\sqrt{2}x}{\ell}\right)} + \frac{4q^2}{4\pi\varepsilon_0\ell} = 4\frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\ell\sqrt{2}}\left[\frac{1}{\left(1+\frac{\sqrt{2}x}{\ell}\right)} + \frac{1}{\left(1-\frac{\sqrt{2}x}{\ell}\right)}\right] + \frac{4q^2}{4\pi\varepsilon_0\ell} =$$

$$= 4\frac{mv^2}{2} + \frac{q^2}{4\pi\varepsilon_0\ell\sqrt{2}}\left[\frac{1-\frac{\sqrt{2}x}{\ell}+1+\frac{\sqrt{2}x}{\ell}}{\left(1-\frac{2x^2}{\ell^2}\right)}\right] + \frac{4q^2}{4\pi\varepsilon_0\ell} = 4\frac{mv^2}{2} + \frac{2q^2}{4\pi\varepsilon_0\ell\sqrt{2}}\left(\frac{1}{1-\frac{2x^2}{\ell^2}}\right) + \frac{4q^2}{4\pi\varepsilon_0\ell} \approx$$

$$\approx 4\frac{mv^2}{2} + \frac{2q^2}{4\pi\varepsilon_0\ell\sqrt{2}}\left(1+\frac{2x^2}{\ell^2}\right) + \frac{4q^2}{4\pi\varepsilon_0\ell} = 4\frac{mv^2}{2} + \frac{8q^2}{4\pi\varepsilon_0\ell^3\sqrt{2}}\frac{x^2}{2} + \frac{2q^2}{4\pi\varepsilon_0\ell\sqrt{2}} + \frac{4q^2}{4\pi\varepsilon_0\ell} \Rightarrow \omega^2 = \frac{k}{m} = \frac{\frac{8q^2}{4\pi\varepsilon_0\ell^3\sqrt{2}}}{4m} = \frac{2q^2}{4\pi\varepsilon_0 m\ell^3\sqrt{2}}$$

$$\mathcal{E} = \frac{A\dot{z}^2}{2} + \frac{Bz^2}{2} + Fz + C \Rightarrow \omega^2 = \frac{B}{A}$$



$$v_x = \frac{vx}{\ell} \Rightarrow v_x = \frac{vx^2}{\ell^2} \Rightarrow v_x = \frac{vx^n}{\ell^n}$$

$$dM_x = \frac{Mdx}{\ell} \Rightarrow$$

$$d\mathcal{E}_{kx} = \frac{dM_x v_x{}^2}{2} \Rightarrow \mathcal{E}_{kM} = \int_0^\ell \frac{1}{2}\frac{M}{\ell}\frac{v^2x^2}{\ell^2}dx = \int_0^\ell \frac{Mv^2x^2dx}{2\ell^3} = \frac{Mv^2}{2\ell^3}\frac{x^3}{3}\Big|_0^\ell = \frac{Mv^2}{2\ell^3}\frac{\ell^3}{3} = \frac{M}{3}\frac{v^2}{2}$$

$$\mathcal{E} = \frac{M}{3}\frac{v^2}{2} + \frac{mv^2}{2} + \frac{kx^2}{2} = \left(m + \frac{M}{3}\right)\frac{v^2}{2} + \frac{kx^2}{2} \Rightarrow \omega^2 = \frac{k}{m + \frac{M}{3}} = \frac{3k}{M + 3m}$$

$$f = u.v \Rightarrow d(u.v) = du.v + u.dv$$

$$\int d(u.v) = \int du.v + \int u.dv \Rightarrow uv = \int du.v + \int u.dv \Rightarrow \int u.dv = u.v - \int v.du$$

$$\int x \sin x dx = -\int x d(\cos x) = x.(-\cos x) - \int(-\cos x)dx = -x\cos x + \sin x + C = f(x)$$

$$f'(x) = -\cos x + x\sin x + \cos x = x\sin x$$

$$\int x^2 \ln x dx = \int \ln x d\left(\frac{x^3}{3}\right) = \frac{x^3}{3}.\ln x - \int \frac{x^3}{3}.\frac{1}{x}dx = \frac{x^3}{3}.\ln x - \int \frac{x^2dx}{3} = \frac{x^3}{3}.\ln x - \frac{x^3}{9} +$$

$$\sqrt{a^2 - x^2} \Rightarrow x = a\sin z \Rightarrow dx = a\cos z dz \Rightarrow \sqrt{a^2 - a^2\sin^2 z} = a\cos z$$

$$\sqrt{x^2 - a^2} \Rightarrow x = \frac{a}{\sin z} \Rightarrow dx = -\frac{a\cos z dz}{\sin^2 z} \Rightarrow \sqrt{\frac{a^2}{\sin^2 z} - a^2} = \sqrt{\frac{a^2(1 - \sin^2 z)}{\sin^2 z}} = \frac{a\cos z}{\sin z} = a\cot z$$

$$\sqrt{a^2 + x^2} = x = a\tan z \Rightarrow dx = \frac{adz}{\cos^2 z} \Rightarrow \sqrt{a^2 + a^2\tan^2 z} = a\sqrt{1 + \frac{\sin^2 z}{\cos^2 z}} = \frac{a}{\cos z}\sqrt{\sin^2 z + \cos^2 z} = \frac{a}{\cos z}$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \frac{a\cos z dz}{\sqrt{a^2 - a^2\sin^2 z}} = \int \frac{a\cos z dz}{a\sqrt{1 - \sin^2 z}} = \int \frac{\cos z dz}{\cos z} = \int dz = z + C = \arcsin\frac{x}{a} + C \Rightarrow \left(\arcsin\frac{x}{a}\right)' = \frac{1}{\sqrt{a^2 - x^2}}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}} \Rightarrow (\arccos x)' = -\frac{1}{\sqrt{1 - x^2}}$$

$$\int \frac{dx}{a^2 + x^2} = \int \frac{\frac{adz}{\cos^2 z}}{a^2 + a^2\tan^2 z} = \int \frac{adz}{a^2\left(1 + \frac{\sin^2 z}{\cos^2 z}\right)\cos^2 z} = \int \frac{dz}{a\left(\frac{\sin^2 z + \cos^2 z}{\cos^2 z}\right)\cos^2 z} = \int \frac{dz}{a} = \frac{z}{a} + C = \frac{1}{a}\arctan\frac{x}{a} + C$$

$$\int \frac{dx}{\sqrt{1+x^2}} = \int \frac{\frac{dz}{\cos^2 z}}{\sqrt{1+\tan^2 z}} = \int \frac{dz}{\cos^2 z.\sqrt{1+\frac{\sin^2 z}{\cos^2 z}}} = \int \frac{dz}{\cos^2 z.\sqrt{\left(\frac{\sin^2 z+\cos^2 z}{\cos^2 z}\right)}} =$$

$$= \int \frac{dz}{\cos^2 z.\frac{1}{\cos z}} = \int \frac{dz}{\cos z} = \int \frac{\cos z dz}{\cos^2 z} = \int \frac{d(\sin z)}{1-\sin^2 z} = \int \frac{dt}{1-t^2} = \int \frac{dt}{(1-t)(1+t)} = \int \frac{\left[(1-t)+(1+t)\right]dt}{2(1-t)(1+t)} =$$

$$= \int \frac{(1-t)dt}{2(1-t)(1+t)} + \int \frac{(1+t)dt}{2(1-t)(1+t)} = \int \frac{dt}{2(1+t)} + \int \frac{dt}{2(1-t)} = \int \frac{d(1+t)}{2(1+t)} - \int \frac{d(1-t)}{2(1-t)} =$$

$$= \frac{1}{2}\ln(1+t) - \frac{1}{2}\ln(1-t) = \frac{1}{2}\ln\frac{1+t}{1-t} = \frac{1}{2}\ln\frac{1+\sin z}{1-\sin z} = \frac{1}{2}\ln\frac{1+\frac{\tan z}{\sqrt{1+\tan^2 z}}}{1-\frac{\tan z}{\sqrt{1+\tan^2 z}}} = \frac{1}{2}\ln\frac{\sqrt{1+\tan^2 z}+\tan z}{\sqrt{1+\tan^2 z}-\tan z} =$$

$$= \frac{1}{2}\ln\frac{\left(\sqrt{1+\tan^2 z}+\tan z\right)^2}{\left(\sqrt{1+\tan^2 z}-\tan z\right)\left(\sqrt{1+\tan^2 z}+\tan z\right)} = \frac{1}{2}\ln\frac{\left(\sqrt{1+\tan^2 z}+\tan z\right)^2}{1+\tan^2 z-\tan^2 z} = \frac{1}{2}\ln\left(\sqrt{1+\tan^2 z}+\tan z\right)^2 =$$

$$= \ln\left(\sqrt{1+\tan^2 z}+\tan z\right) = \ln\left(\sqrt{1+x^2}+x\right)+C$$

$$\left[\ln\left(\sqrt{1+x^2}+x\right)+C\right]' = \frac{1}{\sqrt{1+x^2}+x}.\left(\sqrt{1+x^2}+x\right)' = \frac{1}{\sqrt{1+x^2}+x}.\left(\frac{1}{2\sqrt{1+x^2}}.2x+1\right) =$$

$$= \frac{1}{\sqrt{1+x^2}+x}.\frac{x+\sqrt{1+x^2}}{\sqrt{1+x^2}} = \frac{1}{\sqrt{1+x^2}}$$

$$x = \frac{1}{\sin z} \Rightarrow dx = \left[(\sin z)^{-1}\right]' = -(\sin z)^{-2}(\sin z)' = -\frac{\cos z dz}{\sin^2 z}$$

$$\int \sqrt{x^2-1}\,dx = \int \sqrt{\frac{1}{\sin^2 z}-1}\left(-\frac{\cos z dz}{\sin^2 z}\right) = -\int \frac{\cos z}{\sin^2 z}\sqrt{\frac{1-\sin^2 z}{\sin^2 z}}\,dz = -\int \frac{\cos^2 z}{\sin^3 z}\,dz=$$

$$= -\int \frac{1-\sin^2 z}{\sin^3 z}\,dz = -\int \frac{dz}{\sin^3 z} + \int \frac{dz}{\sin z} =$$

$$-\int \frac{dz}{\sin^3 z} = -\int \frac{\sin z dz}{\sin^4 z} = \int \frac{d(\cos z)}{\left(1-\cos^2 z\right)^2} = \int \frac{dt}{\left(1-t^2\right)^2} = \int \frac{\left(1-t^2+t^2\right)dt}{\left(1-t^2\right)^2} = \int \frac{\left(1-t^2\right)dt}{\left(1-t^2\right)^2} + \int \frac{t^2 dt}{\left(1-t^2\right)^2}$$

$$\int \frac{\left(1-t^2\right)dt}{\left(1-t^2\right)^2} = \int \frac{dt}{1-t^2} = \int \frac{\left[(1-t)+(1+t)\right]dt}{2(1-t)(1+t)} = \int \frac{(1-t)dt}{2(1-t)(1+t)} + \int \frac{(1+t)dt}{2(1-t)(1+t)} =$$

$$= \int \frac{dt}{2(1+t)} + \int \frac{dt}{2(1-t)} = \int \frac{d(1+t)}{2(1+t)} - \int \frac{d(1-t)}{2(1-t)} = \frac{1}{2}\ln(1+t) - \frac{1}{2}\ln(1-t) = \frac{1}{2}\ln\frac{1+t}{1-t} = \frac{1}{2}\ln\frac{1+\cos z}{1-\cos z} =$$

$$= \frac{1}{2}\ln\frac{1+\sqrt{1-\sin^2 z}}{1-\sqrt{1-\sin^2 z}} = \frac{1}{2}\ln\frac{1+\sqrt{1-\frac{1}{x^2}}}{1-\sqrt{1-\frac{1}{x^2}}} = \frac{1}{2}\ln\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}} = \frac{1}{2}\ln\frac{\left(x+\sqrt{x^2-1}\right)^2}{\left(x-\sqrt{x^2-1}\right)\left(x+\sqrt{x^2-1}\right)} =$$

$$= \frac{1}{2}\ln\frac{\left(x+\sqrt{x^2-1}\right)^2}{x^2-x^2+1} = \ln\left(x+\sqrt{x^2-1}\right)$$

$$\int \frac{dt}{\left(1-t^2\right)^2} = \int \frac{\left(1-t^2+t^2\right)dt}{\left(1-t^2\right)^2} = \int \frac{\left(1-t^2\right)dt}{\left(1-t^2\right)^2} + \int \frac{t^2dt}{\left(1-t^2\right)^2} \Rightarrow \int udv = uv - \int vdu \Rightarrow \int \frac{dz}{z^2} = -\frac{1}{z}$$

$$\int \frac{t^2dt}{\left(1-t^2\right)^2} = -\int \frac{td\left(1-t^2\right)}{2\left(1-t^2\right)^2} = \frac{t}{2\left(1-t^2\right)} - \int \frac{dt}{2\left(1-t^2\right)}$$

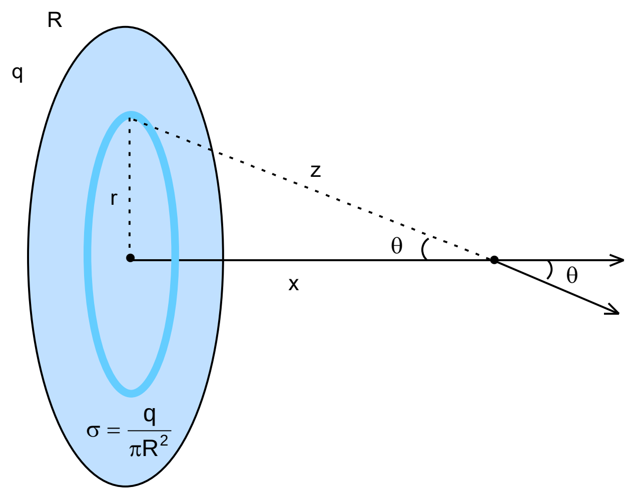
$$\int \frac{dt}{\left(1-t^2\right)^2} = \frac{\cos z}{2\left(1-\cos^2 z\right)} - \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + \ln\left(x+\sqrt{x^2-1}\right) = \frac{\sqrt{1-\sin^2 z}}{2\left(1-1+\sin^2 z\right)} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} =$$

$$= \frac{\sqrt{1-\frac{1}{x^2}}}{2\frac{1}{x^2}} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} = \frac{\sqrt{x^2-1}}{2.x.\frac{1}{x^2}} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} = \frac{x\sqrt{x^2-1}}{2} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2}$$

$$\int \frac{dz}{\sin z} = \int \frac{\sin z dz}{\sin^2 z} = -\int \frac{d(\cos z)}{1-\cos^2 z} = -\int \frac{dt}{1-t^2} = -\ln\left(x+\sqrt{x^2-1}\right)$$

$$\frac{x\sqrt{x^2-1}}{2} + \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} - \ln\left(x+\sqrt{x^2-1}\right) = \frac{x\sqrt{x^2-1}}{2} - \frac{\ln\left(x+\sqrt{x^2-1}\right)}{2} + C$$

$$\int \frac{dz}{\sin z} = \int \frac{dz}{2\sin\frac{z}{2}\cos\frac{z}{2}} = \int \frac{dz}{2\frac{\sin\frac{z}{2}}{\cos\frac{z}{2}}\cos^2\frac{z}{2}} = \int \frac{d\left(\tan\frac{z}{2}\right)}{\tan\frac{z}{2}} = \ln\left(\tan\frac{z}{2}\right) = \ln\left(\frac{\sin\frac{z}{2}}{\cos\frac{z}{2}}\right) = \ln\left(\frac{\sin\frac{z}{2}}{\sqrt{1-\sin^2\frac{z}{2}}}\right)$$



$$dq = 2\pi r\sigma dr \Rightarrow dE = \frac{dq}{4\pi\varepsilon_0 z^2} \Rightarrow dE_x = \frac{dq}{4\pi\varepsilon_0 z^2}.\cos\theta = \frac{dq}{4\pi\varepsilon_0 z^2}.\frac{x}{z} = \frac{2\pi\sigma xrdr}{4\pi\varepsilon_0\left(\sqrt{x^2+r^2}\right)^3}$$

$$E_x = \int_0^R \frac{2\pi\sigma xrdr}{4\pi\varepsilon_0\left(\sqrt{x^2+r^2}\right)^3} = \frac{2\pi\sigma x}{4\pi\varepsilon_0}\int_0^R \frac{rdr}{\left(\sqrt{x^2+r^2}\right)^3} = \frac{\sigma x}{2\varepsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right) = \frac{\sigma}{2\varepsilon_0}\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

$$\int_0^R \frac{rdr}{\left(\sqrt{x^2+r^2}\right)^3} = \int_0^R \frac{d\left(x^2+r^2\right)}{2\left(\sqrt{x^2+r^2}\right)^3} = \int_0^R \frac{d\left(x^2+r^2\right)}{2\left(x^2+r^2\right)^{\frac{3}{2}}} = \frac{1}{2}\int_0^R z^{-\frac{3}{2}}dz = \frac{1}{2}(-2)z^{-\frac{1}{2}} = -\frac{1}{\sqrt{x^2+r^2}}\Bigg|_0^R = -\left(\frac{1}{\sqrt{x^2+R^2}} - \frac{1}{x}\right) =$$

$$= \frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}$$