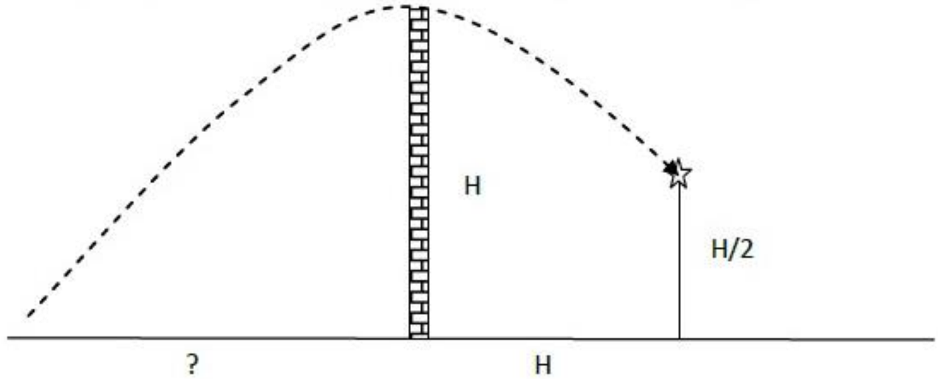
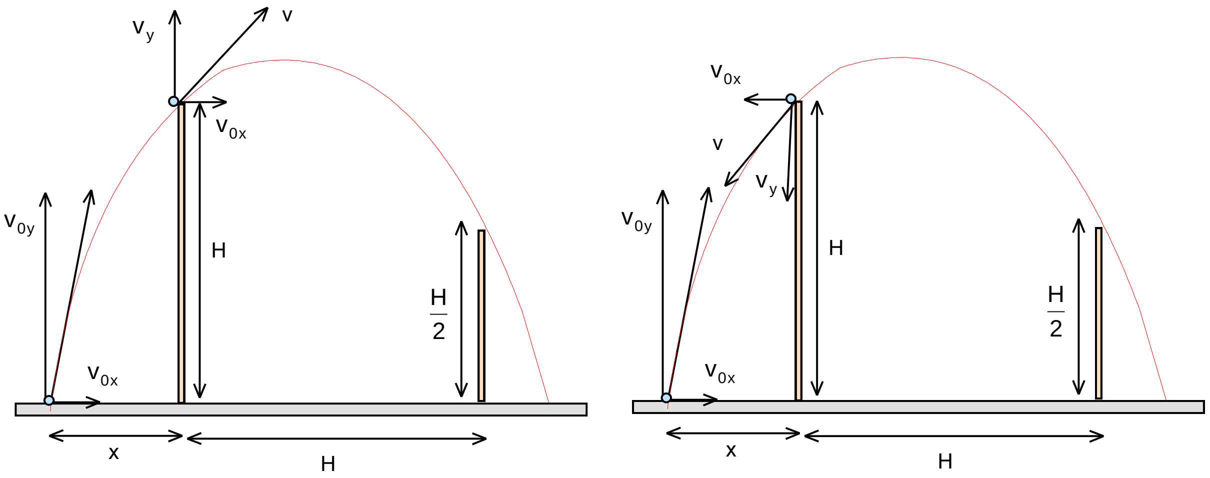
H yüksekliğinde bir duvarın arkasında, duvara H uzaklıkta ve yerden H/2 yükseklikte bir hedef vardır. Bu hedefi duvarın öbür tarafından yapacağımız bir atış ile vurmak istiyoruz. Yapacağımız atışın enerjisinin (yani ilk andaki kinetik enerjisinin) minimum olmasını istiyorsak duvardan ne kadar uzakta durmalıyız? (Duvarın kalınlığı, hava sürtünmesi ihmal edilebilir. Atış yerden yapılmaktadır, atışın açısı serbestçe ayarlanabir, yerçekimi ivmesi g dir)





$$0 = H - v_y t_1 - \frac{g t_1^2}{2} \Rightarrow t_1 = \frac{x}{v_{0x}} \Rightarrow H - \frac{x v_y}{v_{0x}} - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2 = H - \frac{x v_y}{v_{0x}} - \frac{g x^2}{2 v_{0x}^2} = 0 \Rightarrow \frac{x v_y}{v_{0x}} = H - \frac{g x^2}{2 v_{0x}^2}$$

$$\frac{H}{2} = H + v_y t_2 - \frac{g t_2^2}{2} \Rightarrow t_2 = \frac{H}{v_{0x}} \Rightarrow 0 = \frac{H}{2} + \frac{H v_y}{v_{0x}} - \frac{g}{2}\left(\frac{H}{v_{0x}}\right)^2 = \frac{H}{2} + \frac{H v_y}{v_{0x}} - \frac{g H^2}{2 v_{0x}^2} \Rightarrow \frac{H v_y}{v_{0x}} = \frac{g H^2}{2 v_{0x}^2} - \frac{H}{2}$$

$$\frac{\frac{x v_y}{v_{0x}}}{\frac{H v_y}{v_{0x}}} = \frac{H - \frac{g x^2}{2 v_{0x}^2}}{\frac{g H^2}{2 v_{0x}^2} - \frac{H}{2}} \Rightarrow \frac{x}{H} = \frac{2 v_{0x}^2 H - g x^2}{g H^2 - H v_{0x}^2}$$

$$g H^2 x - v_{0x}^2 x H = 2 v_{0x}^2 H^2 - g x^2 H \Rightarrow v_{0x}^2 (x + 2H) = g x (x + H) \Rightarrow v_{0x}^2 = \frac{g x (x + H)}{x + 2H}$$

$$v_y = \frac{g H}{2 v_{0x}} - \frac{v_{0x}}{2}$$

$$v^2 = v_{0x}^2 + v_y^2 = v_{0x}^2 + \left(\frac{g H}{2 v_{0x}} - \frac{v_{0x}}{2}\right)^2 = v_{0x}^2 + \left(\frac{g^2 H^2}{4 v_{0x}^2} - \frac{g H}{2} + \frac{v_{0x}^2}{4}\right) = \frac{g^2 H^2}{4 v_{0x}^2} - \frac{g H}{2} + \frac{5 v_{0x}^2}{4}$$

$$v_0^2 = v^2 + 2gH = \frac{g^2 H^2}{4 v_{0x}^2} - \frac{g H}{2} + \frac{5 v_{0x}^2}{4} + 2gH = \frac{g^2 H^2}{4 v_{0x}^2} + \frac{3 g H}{2} + \frac{5 v_{0x}^2}{4} = \frac{g^2 H^2}{4 \frac{g x (x + H)}{x + 2H}} + \frac{3 g H}{2} + \frac{5}{4} \frac{g x (x + H)}{x + 2H} =$$

$$= \frac{g H^2 (x + 2H)}{4 x (x + H)} + \frac{5 g x (x + H)}{4 (x + 2H)} + \frac{3 g H}{2} = \frac{g H^2 (x + 2H)^2 + 5 g x^2 (x + H)^2 + 6 g x H (x + H)(x + 2H)}{4 x (x + H)(x + 2H)}$$

$$v_0{}^2=\frac{gH^2\left(x+2H\right)^2+5gx^2\left(x+H\right)^2+6gxH\left(x+H\right)\left(x+2H\right)}{4x\left(x+H\right)\left(x+2H\right)}=$$

$$g\frac{H^2\left(x^2+4Hx+4H^2\right)+5x^2\left(x^2+2Hx+H^2\right)+6xH\left(x^2+3Hx+2H^2\right)}{4x\left(x+H\right)\left(x+2H\right)}=$$

$$=g\frac{x^2H^2+4H^3x+4H^4+5x^4+10Hx^3+5H^2x^2+6Hx^3+18H^2x^2+12H^3x}{4x\left(x+H\right)\left(x+2H\right)}=$$

$$g\frac{24x^2H^2+16H^3x+4H^4+5x^4+16Hx^3}{4x\left(x+H\right)\left(x+2H\right)}$$

$$E_{k0}=\frac{mv_0{}^2}{2}=\frac{mg}{8}\frac{24x^2H^2+16H^3x+4H^4+5x^4+16Hx^3}{x\left(x+H\right)\left(x+2H\right)}$$

$$v_0{}^2=\frac{g^2H^2}{4v_{0x}{}^2}+\frac{3gH}{2}+\frac{5v_{0x}{}^2}{4}$$

$$\frac{d}{dv_{0x}}\left(v_0{}^2\right)=-\frac{2.g^2H^2}{4v_{0x}{}^3}+\frac{2.5v_{0x}}{4}=0\Rightarrow\frac{g^2H^2}{v_{0x}{}^4}=5\Rightarrow v_{0x}=\frac{\sqrt{gH}}{\sqrt[4]{5}}\Rightarrow v_{0x}{}^2=\frac{gH}{\sqrt{5}}$$

$$v_{0x}=0,6687\sqrt{gH}\Rightarrow v_{0x}{}^2=0,4472gH$$

$$v_y=\frac{gH}{2.0,6687\sqrt{gH}}-\frac{0,6687\sqrt{gH}}{2}=0,7477\sqrt{gH}-0,3344\sqrt{gH}=0,4133\sqrt{gH}$$

$$\frac{x.0,4133\sqrt{gH}}{0,6687\sqrt{gH}}=H-\frac{gx^2}{2.0,4472gH}\Rightarrow 0,618x=H-\frac{1,118x^2}{H}\Rightarrow$$

$$,618Hx=H^2-1,118x^2\Rightarrow 1,118x^2+0,618Hx-H^2=0$$

$$x=\frac{-0,618H+\sqrt{(0,618H)^2+4.1,118H^2}}{2.1.118}=0,7089H=0,7H$$

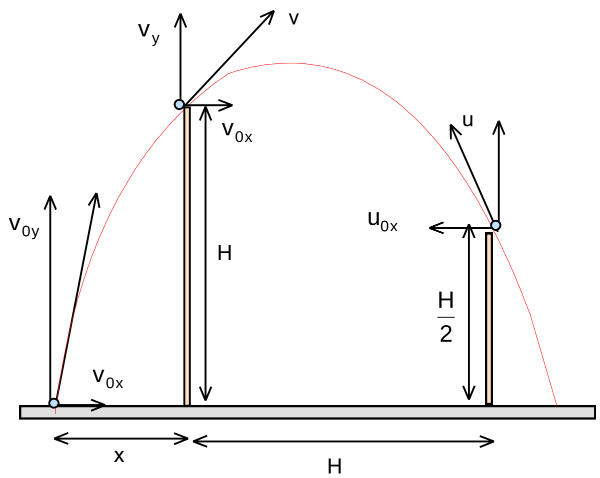
$$v_0{}^2=\frac{g^2H^2}{4.0,4472gH}+\frac{3gH}{2}+\frac{5.0,4472gH}{4}=\frac{gH}{4.0,4472}+\frac{3gH}{2}+\frac{5.0,4472gH}{4}=$$

$$=\left(0,559+1,5+0,559\right)gH=2,618gH$$

$$E_{k0}=\frac{m}{2}.2,618gH=1,3mgH$$

$$E_{k0}=\frac{mg}{8}\frac{24\left(0,7H\right)^2H^2+16H^3.0,7H+4H^4+5\left(0,7H\right)^4+16H\left(0,7H\right)^3}{0,7H\left(0,7H+H\right)\left(0,7H+2H\right)}=$$

$$=\frac{mgH}{8}\frac{24.0,49+16.0,7+4+5.0,24+16.0,343}{0,7.1,7.2,7}=\frac{mgH}{8}\frac{11,76+11,2+4+1,2+5,49}{3,213}=\frac{mgH}{8}\frac{33,65}{3,213}=1,3mgH$$



$$H=\frac{H}{2}+u_{0y}t-\frac{gt^2}{2}\Rightarrow t=\frac{H}{u_{0x}}\Rightarrow 0=-\frac{H}{2}+\frac{Hu_{0y}}{u_{0x}}-\frac{g}{2}\left(\frac{H}{u_{0x}}\right)^2=-\frac{H}{2}+\frac{Hu_{0y}}{u_{0x}}-\frac{gH^2}{2u_{0x}{}^2}=0\Rightarrow\frac{Hu_{0y}}{u_{0x}}=\frac{H}{2}+\frac{gH^2}{2u_{0x}{}^2}$$

$$u_{0y}=\frac{u_{0x}}{2}+\frac{gH}{2u_{0x}}$$

$$u_0{}^2=u_{0x}{}^2+u_{0y}{}^2=u_{0x}{}^2+\left(\frac{u_{0x}}{2}+\frac{gH}{2u_{0x}}\right)^2=u_{0x}{}^2+\left(\frac{g^2H^2}{4u_{0x}{}^2}+\frac{gH}{2}+\frac{u_{0x}{}^2}{4}\right)=\frac{g^2H^2}{4u_{0x}{}^2}+\frac{gH}{2}+\frac{5u_{0x}{}^2}{4}$$

$$v_0{}^2=v^2+2g.\frac{H}{2}=\frac{g^2H^2}{4u_{0x}{}^2}+\frac{gH}{2}+\frac{5u_{0x}{}^2}{4}+gH=\frac{g^2H^2}{4u_{0x}{}^2}+\frac{3gH}{2}+\frac{5u_{0x}{}^2}{4}$$

$$\frac{d}{dv_{0x}}\left(v_0{}^2\right)=-\frac{2.g^2H^2}{4v_{0x}{}^3}+\frac{2.5v_{0x}}{4}=0\Rightarrow\frac{g^2H^2}{v_{0x}{}^4}=5\Rightarrow v_{0x}=\frac{\sqrt{gH}}{\sqrt[4]{5}}\Rightarrow v_{0x}{}^2=\frac{gH}{\sqrt{5}}$$

$$v_{0x}=0,6687\sqrt{gH}\Rightarrow v_{0x}{}^2=0,4472gH$$

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$$\frac{x.0,4133\sqrt{gH}}{0,6687\sqrt{gH}}=H-\frac{gx^2}{2.0,4472gH}\Rightarrow 0,618x=H-\frac{1,118x^2}{H}$$

$$0,618Hx=H^2-1,118x^2\Rightarrow 1,118x^2+0,618Hx-H^2=0$$

$$x=\frac{-0,618H+\sqrt{\left(0,618H\right)^2+4.1,118H^2}}{2.1.118}=0,7089H=0,7H$$