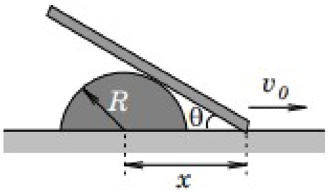


Şekil 1

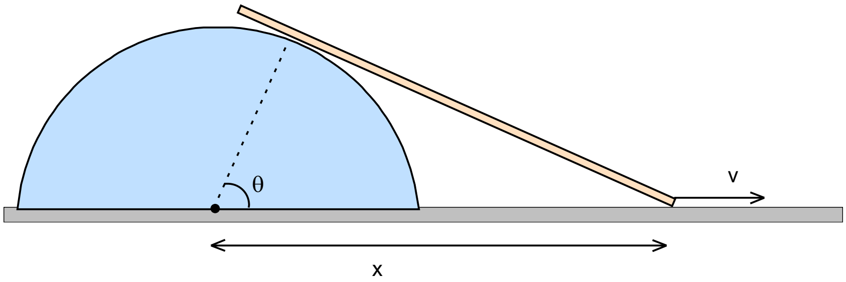
Alıştırma 5.2 ✧

İnce bir kalas, yarıçapı R olan bir yarım silindire dayalı durmaktadır (Şekil 1). Kalasın alt ucu sabit v_0 hızıyla sağa doğru hareket etmeye başlıyor. Her türlü sürtünmenin ihmal edildiği sistemde; kalasın açısal hızını ve ivmesini, kalasın alt ucunun silindirin merkezine olan uzaklığının, x, fonksiyonu olarak bulunuz.*

Cevap

$$w = \frac{Rv_0}{x\sqrt{x^2 - R^2}}$$

$$\alpha = \frac{Rv_0^2(R^2 - 2x^2)}{x^2(x^2 - R^2)^{3/2}}$$



$$x = \frac{R}{\cos\theta} \Rightarrow \sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{R^2}{x^2}} = \frac{\sqrt{x^2-R^2}}{x} \Rightarrow \frac{df}{dx} = \frac{df}{dy}.\frac{dy}{dx}$$

$$dx = R\frac{d}{d\theta}\left[(\cos\theta)^{-1}\right] = -R(\cos\theta)^{-2}.(-\sin\theta)d\theta = \frac{R\sin\theta}{\cos^2\theta}d\theta = \frac{R\frac{\sqrt{x^2-R^2}}{x}}{\frac{R^2}{x^2}}d\theta = \frac{x\sqrt{x^2-R^2}}{R}d\theta$$

$$\frac{dx}{dt} = \frac{x\sqrt{x^2-R^2}}{R}\frac{d\theta}{dt} \Rightarrow v = \frac{x\sqrt{x^2-R^2}}{R}\omega \Rightarrow \omega = \frac{vR}{x\sqrt{x^2-R^2}}$$

$$x+\Delta x = \frac{R}{\cos(\theta+\Delta\theta)} = \frac{R}{\cos\theta\cos\Delta\theta - \sin\theta\sin\Delta\theta} \approx \frac{R}{\cos\theta - \sin\theta.\Delta\theta} = \frac{R}{\cos\theta\left(1-\frac{\sin\theta.\Delta\theta}{\cos\theta}\right)} \approx \frac{R}{\cos\theta}\left(1+\frac{\sin\theta.\Delta\theta}{\cos\theta}\right) = \frac{R}{\cos\theta} + \frac{R\sin\theta.\Delta\theta}{\cos^2\theta}$$

$$v = \frac{(x+\Delta x)-x}{\Delta t} = \frac{1}{\Delta t}.\left[\left(\frac{R}{\cos\theta} + \frac{R\sin\theta.\Delta\theta}{\cos^2\theta}\right) - \frac{R}{\cos\theta}\right] = \frac{R\sin\theta.\Delta\theta}{\Delta t\cos^2\theta} = \frac{R\sin\theta}{\cos^2\theta}\omega$$

$$\frac{d\omega}{dt} = \frac{d}{dx}\left(\frac{vR}{x\sqrt{x^2-R^2}}\right).\frac{dx}{dt} = v.\frac{d}{dx}\left(\frac{vR}{x\sqrt{x^2-R^2}}\right) = v^2R\frac{d}{dx}\left[x^{-1}(x^2-R^2)^{-\frac{1}{2}}\right] = v^2R\left[-x^{-2}(x^2-R^2)^{-\frac{1}{2}} - x^{-1}.\frac{1}{2}.(x^2-R^2)^{-\frac{3}{2}}.2x\right] =$$

$$= v^2R\left[-\frac{1}{x^2\sqrt{x^2-R^2}} - \frac{1}{\sqrt{(x^2-R^2)^3}}\right] = -\frac{v^2R}{\sqrt{x^2-R^2}}\left(\frac{1}{x^2} + \frac{1}{x^2-R^2}\right) = -\frac{v^2R}{\sqrt{x^2-R^2}}\frac{2x^2-R^2}{x^2(x^2-R^2)} = -\frac{v^2R(2x^2-R^2)}{x^2\sqrt{(x^2-R^2)^3}}$$

$$\omega(x)=\frac{vR}{x\sqrt{x^2-R^2}}\Rightarrow\omega(x+\Delta x)=\frac{vR}{(x+\Delta x)\sqrt{(x+\Delta x)^2-R^2}}$$

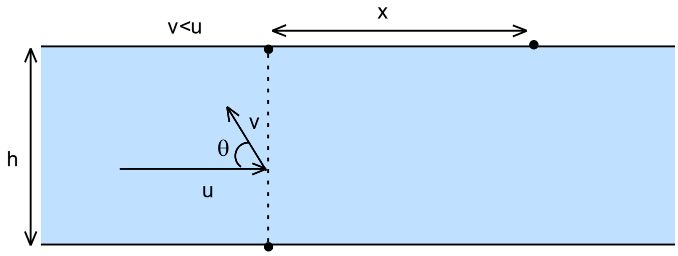
$$\alpha=\frac{\omega(x)-\omega(x+\Delta x)}{\Delta t}=\frac{vR}{\Delta t}\left(\frac{1}{x\sqrt{x^2-R^2}}-\frac{1}{(x+\Delta x)\sqrt{(x+\Delta x)^2-R^2}}\right)=$$

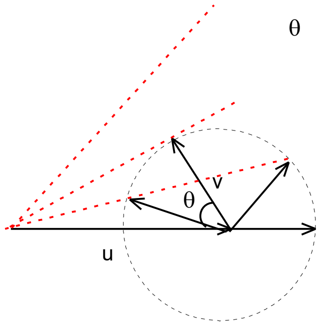
$$\frac{1}{(x+\Delta x)\sqrt{(x+\Delta x)^2-R^2}}=\frac{1}{x\left(1+\frac{\Delta x}{x}\right)\sqrt{x^2\left(1+\frac{\Delta x}{x}\right)^2-R^2}}\approx\frac{1}{x\left(1+\frac{\Delta x}{x}\right)\sqrt{x^2+2x\Delta x+(\Delta x)^2-R^2}}\approx$$

$$\approx\frac{1}{x\left(1+\frac{\Delta x}{x}\right)\sqrt{x^2+2x\Delta x-R^2}}=\frac{1}{x\sqrt{x^2-R^2}\left(1+\frac{\Delta x}{x}\right)\sqrt{1+\frac{2x\Delta x}{x^2-R^2}}}\approx\frac{1}{x\sqrt{x^2-R^2}\left(1+\frac{\Delta x}{x}\right)\left(1+\frac{x\Delta x}{x^2-R^2}\right)}=$$

$$=\frac{1}{x\sqrt{x^2-R^2}\left(1+\frac{\Delta x}{x}+\frac{x\Delta x}{x^2-R^2}+\frac{(\Delta x)^2}{x^2-R^2}\right)}\approx\frac{1}{x\sqrt{x^2-R^2}\left(1+\frac{(2x^2-R^2)\Delta x}{x(x^2-R^2)}\right)}\approx\frac{1}{x\sqrt{x^2-R^2}}\left(1-\frac{(2x^2-R^2)\Delta x}{x(x^2-R^2)}\right)$$

$$\alpha=\frac{vR}{\Delta t}\left[\frac{1}{x\sqrt{x^2-R^2}}-\frac{1}{x\sqrt{x^2-R^2}}\left(1-\frac{(2x^2-R^2)\Delta x}{x(x^2-R^2)}\right)\right]=\frac{vR}{x(2x^2-R^2)}\frac{(2x^2-R^2)}{x(x^2-R^2)}\frac{\Delta x}{\Delta t}=\frac{v^2R(2x^2-R^2)}{x^2\sqrt{(x^2-R^2)^3}}$$

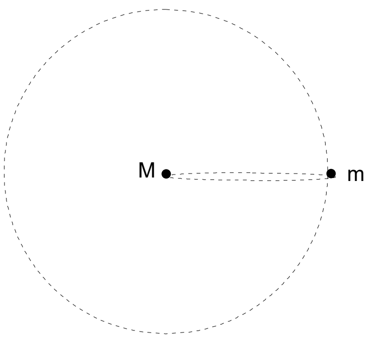




$$t=\frac{h}{v\sin\theta}\Rightarrow x=(u-v\cos\theta)t=\frac{h(u-v\cos\theta)}{v\sin\theta}$$

$$\frac{dx}{d\theta}=\frac{h}{v}\frac{d}{d\theta}\left(\frac{u-v\cos\theta}{\sin\theta}\right)=\frac{h}{v}\cdot\frac{v\sin\theta.\sin\theta-(u-v\cos\theta)\cos\theta}{\sin^2\theta}=\frac{h}{v}\cdot\frac{v\sin^2\theta-u\cos\theta+v\cos^2\theta}{\sin^2\theta}=\frac{h}{v}\cdot\frac{v-u\cos\theta}{\sin^2\theta}=0$$

$$v-u\cos\theta=0\Rightarrow\cos\theta=\frac{v}{u}$$



$$\frac{T_0^2}{R^3}=\frac{T^2}{a^3}\Rightarrow a=\frac{R}{2}$$

$$\frac{T_0^2}{R^3}=\frac{T^2}{\left(\frac{R}{2}\right)^3}\Rightarrow\frac{T_0^2}{R^3}=\frac{8T^2}{R^3}\Rightarrow T_0=2\sqrt{2}T\Rightarrow T=\frac{T_0}{2\sqrt{2}}\Rightarrow t=\frac{T}{2}=\frac{T_0}{4\sqrt{2}}$$