$$ma=-kx-\chi\dot{x}+F_0\sin\Omega t\Rightarrow\ddot{x}+\frac{\chi}{m}\dot{x}+\frac{k}{m}x=\frac{F_0}{m}\sin\Omega t\Rightarrow\ddot{x}+\beta\dot{x}+\omega_0{}^2x=\frac{F_0}{m}\sin\Omega t$$

$$x=A\sin(\Omega t+\delta)\Rightarrow\dot{x}=\Omega A\cos(\Omega t+\delta)\Rightarrow\ddot{x}=-\Omega^2A\sin(\Omega t+\delta)$$

$$-\Omega^2A\sin(\Omega t+\delta)+\beta\Omega A\cos(\Omega t+\delta)+\omega_0{}^2A\sin(\Omega t+\delta)=\frac{F_0}{m}\sin\Omega t$$

$$-\Omega^2A(\sin\Omega t\cos\delta+\cos\Omega t\sin\delta)+\beta\Omega A(\cos\Omega t\cos\delta-\sin\Omega t\sin\delta)+\omega_0{}^2A(\sin\Omega t\cos\delta+\cos\Omega t\sin\delta)=\frac{F_0}{m}\sin\Omega t$$

$$\left(-\Omega^2A\cos\delta-\beta\Omega A\sin\delta+\omega_0{}^2A\cos\delta\right)\sin\Omega t+\left(-\Omega^2A\sin\delta+\beta\Omega A\cos\delta+\omega_0{}^2A\sin\delta\right)\cos\Omega t=\frac{F_0}{m}\sin\Omega t$$

$$-\Omega^2A\sin\delta+\beta\Omega A\cos\delta+\omega_0{}^2A\sin\delta=0\Rightarrow-\Omega^2\sin\delta+\beta\Omega\cos\delta+\omega_0{}^2\sin\delta=0$$

$$-\Omega^2\tan\delta+\beta\Omega+\omega_0{}^2\tan\delta=0\Rightarrow\left(\omega_0{}^2-\Omega^2\right)\tan\delta=-\beta\Omega\Rightarrow\tan\delta=-\frac{\beta\Omega}{\omega_0{}^2-\Omega^2}$$

$$\sin\delta=\frac{\tan\delta}{\sqrt{1+\tan^2\delta}}=\frac{-\frac{\beta\Omega}{\omega_0{}^2-\Omega^2}}{\sqrt{1+\left(-\frac{\beta\Omega}{\omega_0{}^2-\Omega^2}\right)^2}}=-\frac{\beta\Omega}{\left(\omega_0{}^2-\Omega^2\right)\sqrt{1+\frac{\beta^2\Omega^2}{\left(\omega_0{}^2-\Omega^2\right)^2}}}=-\frac{\beta\Omega}{\left(\omega_0{}^2-\Omega^2\right)\frac{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}{\left(\omega_0{}^2-\Omega^2\right)}}=-\frac{\beta\Omega}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}$$

$$\cos\delta=\frac{1}{\sqrt{1+\tan^2\delta}}=\frac{1}{\sqrt{1+\left(-\frac{\beta\Omega}{\omega_0{}^2-\Omega^2}\right)^2}}=\frac{1}{\sqrt{1+\frac{\beta^2\Omega^2}{\left(\omega_0{}^2-\Omega^2\right)^2}}}=\frac{1}{\frac{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}{\omega_0{}^2-\Omega^2}}=\frac{\omega_0{}^2-\Omega^2}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}$$

$$\left(-\Omega^2A\cos\delta-\beta\Omega A\sin\delta+\omega_0{}^2A\cos\delta\right)\sin\Omega t=\frac{F_0}{m}\sin\Omega t\Rightarrow\left(-\Omega^2\cos\delta-\beta\Omega\sin\delta+\omega_0{}^2\cos\delta\right)A=\frac{F_0}{m}$$

$$\left[\left(\omega_0{}^2-\Omega^2\right)\cos\delta-\beta\Omega\sin\delta\right]A=\frac{F_0}{m}\Rightarrow\left[\left(\omega_0{}^2-\Omega^2\right).\frac{\omega_0{}^2-\Omega^2}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}-\beta\Omega.\left(-\frac{\beta\Omega}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}\right)\right]A=\frac{F_0}{m}$$

$$\left[\frac{\left(\omega_0{}^2-\Omega^2\right)^2}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}+\frac{\beta^2\Omega^2}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}\right]A=\frac{F_0}{m}\Rightarrow\frac{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}{\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}A=\frac{F_0}{m}\Rightarrow A\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}=\frac{F_0}{m}$$

$$A=\frac{F_0}{m\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}$$

$$\sum\mathcal{E}+\sum\mathcal{E}_{in}=\sum IR+\sum\frac{q}{C}\Rightarrow-L\frac{dI}{dt}=IR+\frac{q}{C}\Rightarrow\frac{dI}{dt}+\frac{R}{L}I+\frac{1}{LC}q=0\Rightarrow\ddot{q}+\beta\dot{q}+\omega_0{}^2q=0$$

$$U_0\sin\Omega t-L\frac{dI}{dt}=IR+\frac{q}{C}\Rightarrow\ddot{q}+\beta\dot{q}+\omega_0{}^2q=\frac{U_0}{L}\sin\Omega t$$

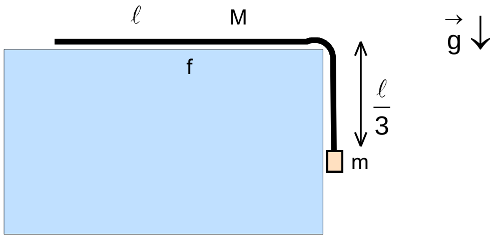
$$A=\frac{F_0}{m\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}\Rightarrow Q=\frac{U_0}{L\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}\Rightarrow x=A\sin(\Omega t+\delta)$$

$$q=Q\sin(\Omega t+\delta)\Rightarrow I=\dot{q}=\Omega Q\cos(\Omega t+\delta)=I_{mak}\cos(\Omega t+\delta)$$

$$I_{mak}=\frac{\Omega U_0}{L\sqrt{\left(\omega_0{}^2-\Omega^2\right)^2+\beta^2\Omega^2}}=\frac{\Omega U_0}{L\sqrt{\left(\frac{1}{LC}-\Omega^2\right)^2+\frac{R^2\Omega^2}{L^2}}}=\frac{\Omega U_0}{\sqrt{L^2\Omega^2\left(\frac{1}{\Omega LC}-\Omega\right)^2+\frac{R^2\Omega^2}{L^2}}}=$$

$$=\frac{\Omega U_0}{\Omega\sqrt{L^2\left(\frac{1}{\Omega LC}-\Omega\right)^2+R^2}}=\frac{U_0}{\sqrt{\left(\frac{L}{\Omega LC}-\Omega L\right)^2+R^2}}=\frac{U_0}{\sqrt{\left(\frac{1}{\Omega C}-\Omega L\right)^2+R^2}}$$

$$Z=\sqrt{\left(\frac{1}{\Omega C}-\Omega L\right)^2+R^2}=\sqrt{\left(\Omega L-\frac{1}{\Omega C}\right)^2+R^2}=\sqrt{\left(X_L-X_C\right)^2+R^2}\Rightarrow X_L=\Omega L\Rightarrow X_C=\frac{1}{\Omega C}$$



$$ma = -kx \Rightarrow \ddot{x} + \frac{k}{m}x = 0 \Rightarrow \ddot{x} + \omega^2 x = 0$$

$$(M+m)a = mg + \frac{Mx}{\ell}.g - f.\frac{M(\ell - x)}{\ell}.g = \frac{M(1+f)x}{\ell} + mg - fMg \Rightarrow \ddot{x} - \frac{M(1+f)x}{\ell(M+m)} = \frac{(m-fM)g}{M+m} \Rightarrow \ddot{x} - \omega^2 x = C$$

$$\omega^2 = \frac{M(1+f)}{\ell(M+m)} \Rightarrow C = \frac{(m-fM)g}{M+m}$$

$$x = Ae^{\lambda t} \Rightarrow \dot{x} = \lambda A e^{\lambda t} \Rightarrow \ddot{x} = \lambda^2 A e^{\lambda t} \Rightarrow \ddot{x} - \omega^2 x = 0 \Rightarrow \lambda^2 Ae^{\lambda t} - \omega^2 Ae^{\lambda t} = 0 \Rightarrow \lambda^2 - \omega^2 = 0 \Rightarrow \lambda^2 = \omega^2 \Rightarrow \lambda = \pm\omega$$

$$\lambda^2 + {\omega_0}^2 = 0 \Rightarrow \lambda^2 = -{\omega_0}^2 \Rightarrow \lambda = \pm i\omega_0$$

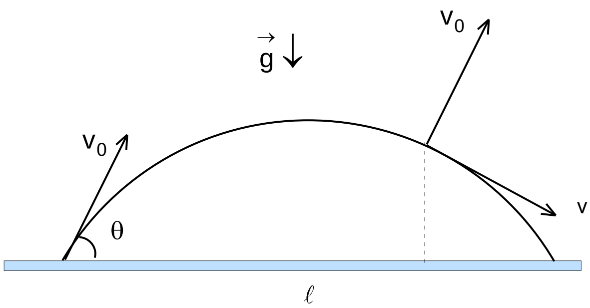
$$x_1 = A_1 e^{\omega t} + A_2 e^{-\omega t} \Rightarrow x_2 = D \Rightarrow \dot{x} = \ddot{x} = 0 \Rightarrow 0 - \omega^2 D = C \Rightarrow D = -\frac{C}{\omega^2}$$

$$x = x_1 + x_2 = A_1 e^{\omega t} + A_2 e^{-\omega t} - \frac{C}{\omega^2} \Rightarrow \dot{x} = \omega A_1 e^{\omega t} - \omega A_2 e^{-\omega t} \Rightarrow x(0) = \frac{\ell}{3} \Rightarrow \dot{x}(0) = 0$$

$$\frac{\ell}{3} = A_1 + A_2 - \frac{C}{\omega^2} \Rightarrow 0 = \omega A_1 - \omega A_2;\ A_1 = A_2 \Rightarrow 2A_1 = \frac{\ell}{3} + \frac{C}{\omega^2} \Rightarrow A_1 = \frac{1}{2}\left(\frac{\ell}{3} + \frac{C}{\omega^2}\right)$$

$$x = \frac{1}{2}\left(\frac{\ell}{3} + \frac{C}{\omega^2}\right)\left(e^{\omega t} + e^{-\omega t}\right) - \frac{C}{\omega^2} \Rightarrow x = \frac{1}{2}\left(\frac{\ell}{3} + \frac{C}{\omega^2}\right)(1+1) - \frac{C}{\omega^2} = \frac{1}{2}\frac{\ell}{3}.2 + \frac{1}{2}\frac{C}{\omega^2}.2 - \frac{C}{\omega^2} = \frac{\ell}{3}$$

$$\ell = \frac{1}{2}\left(\frac{\ell}{3} + \frac{C}{\omega^2}\right)\left(e^{\omega t} + e^{-\omega t}\right) - \frac{C}{\omega^2} \Rightarrow \frac{\ell + \frac{C}{\omega^2}}{\frac{1}{2}\left(\frac{\ell}{3} + \frac{C}{\omega^2}\right)} = e^{\omega t} + e^{-\omega t} = B \Rightarrow e^{\omega t} + \frac{1}{e^{\omega t}} = B \Rightarrow e^{2\omega t} + 1 = Be^{\omega t} \Rightarrow e^{2\omega t} - Be^{\omega t} + 1 = 0 \Rightarrow z^2 - Bz + 1 = 0$$



$$v_{0x} = v_0\cos\theta \Rightarrow v_{0y} = v_0\sin\theta$$

$$v_x = v_0\cos\theta \Rightarrow v_y = v_{0y} - gt = v_0\sin\theta - gt$$

$$\vec{v}_0.\vec{v} = 0 \Rightarrow \vec{a}.\vec{b} = a_x b_x + a_y b_y = ab\cos\theta$$

$$v_0\cos\theta.v_0\cos\theta + v_0\sin\theta.(v_0\sin\theta - gt) = 0 \Rightarrow {v_0}^2\cos^2\theta + {v_0}^2\sin^2\theta - v_0 gt\sin\theta = 0 \Rightarrow t = \frac{v_0}{g\sin\theta}$$