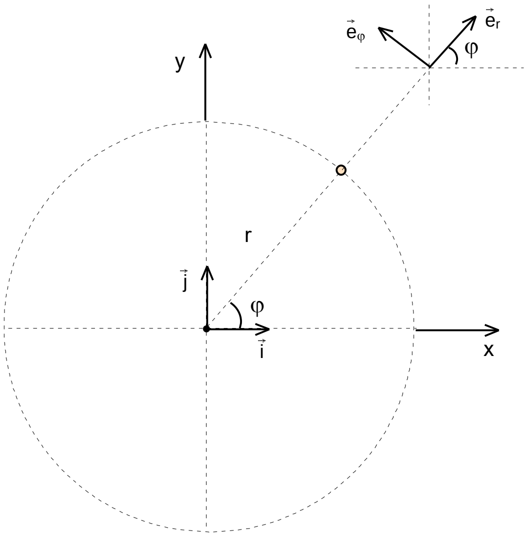


$$e_{rx} = \cos\varphi \Rightarrow e_{ry} = \sin\varphi$$
$$e_{\varphi x} = -\sin\varphi \Rightarrow e_{\varphi y} = \cos\varphi$$

$$x = r\cos\varphi \Rightarrow y = r\sin\varphi$$
$$r = \sqrt{x^2 + y^2} \Rightarrow \varphi = \arctan\frac{y}{x}$$

$$v_x = \dot{x} = \dot{r}\cos\varphi - r\sin\varphi.\dot{\varphi}$$
$$v_y = \dot{y} = \dot{r}\sin\varphi + r\cos\varphi.\dot{\varphi}$$

$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{\left(\dot{r}\cos\varphi - r\sin\varphi.\dot{\varphi}\right)^2 + \left(\dot{r}\sin\varphi + r\cos\varphi.\dot{\varphi}\right)^2} =$$
$$= \sqrt{\dot{r}^2\cos^2\varphi - 2r\dot{r}\sin\varphi\cos\varphi.\dot{\varphi} + r^2\sin^2\varphi.\left(\dot{\varphi}\right)^2 + \dot{r}^2\sin^2\varphi + 2r\dot{r}\sin\varphi\cos\varphi.\dot{\varphi} + r^2\cos^2\varphi.\left(\dot{\varphi}\right)^2} = \sqrt{\dot{r}^2 + r^2.\left(\dot{\varphi}\right)^2}$$
$$\dot{r} = 0 \Rightarrow v_\varphi = \omega r$$

$$a_x = \ddot{x} = \ddot{r}\cos\varphi - \dot{r}\sin\varphi.\dot{\varphi} - \dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.\left(\dot{\varphi}\right)^2 - r\sin\varphi.\ddot{\varphi} = \ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.\left(\dot{\varphi}\right)^2 - r\sin\varphi.\ddot{\varphi}$$
$$a_y = \ddot{y} = \ddot{r}\sin\varphi + \dot{r}\cos\varphi.\dot{\varphi} + \dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.\left(\dot{\varphi}\right)^2 + r\cos\varphi.\ddot{\varphi} = \ddot{r}\sin\varphi + 2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.\left(\dot{\varphi}\right)^2 + r\cos\varphi.\ddot{\varphi}$$
$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{\left(\ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.\left(\dot{\varphi}\right)^2 - r\sin\varphi.\ddot{\varphi}\right)^2 + \left(2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.\left(\dot{\varphi}\right)^2 + r\cos\varphi.\ddot{\varphi}\right)^2} =$$

$$\left[(a+b)+(c+d)\right]^2 = (a+b)^2 + 2(a+b)(c+d) + (c+d)^2 = a^2 + 2ab + b^2 + 2ac + 2ad + 2bc + 2bd + c^2 + 2cd + d^2 =$$
$$a^2 + b^2 + c^2 + d^2 + 2ab + 2ac + 2ad + 2bc + 2bd + 2cd$$

$$a^2 + b^2 + c^2 + d^2 + 2ab + 2ac + 2ad + 2bc + 2bd + 2cd$$
$$a_x = \ddot{x} = \ddot{r}\cos\varphi - \dot{r}\sin\varphi.\dot{\varphi} - \dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.\left(\dot{\varphi}\right)^2 - r\sin\varphi.\ddot{\varphi} = \ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.\left(\dot{\varphi}\right)^2 - r\sin\varphi.\ddot{\varphi}$$
$$a_y = \ddot{y} = \ddot{r}\sin\varphi + \dot{r}\cos\varphi.\dot{\varphi} + \dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.\left(\dot{\varphi}\right)^2 + r\cos\varphi.\ddot{\varphi} = \ddot{r}\sin\varphi + 2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.\left(\dot{\varphi}\right)^2 + r\cos\varphi.\ddot{\varphi}$$
$$a = \sqrt{{a_x}^2 + {a_y}^2} = \sqrt{\left(\ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.\left(\dot{\varphi}\right)^2 - r\sin\varphi.\ddot{\varphi}\right)^2 + \left(\ddot{r}\sin\varphi + 2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.\left(\dot{\varphi}\right)^2 + r\cos\varphi.\ddot{\varphi}\right)^2} =$$
$$\left(\ddot{r}\cos\varphi\right)^2 + \left(2\dot{r}\sin\varphi.\dot{\varphi}\right)^2 + \left[r\cos\varphi.\left(\dot{\varphi}\right)^2\right]^2 + \left(r\sin\varphi.\ddot{\varphi}\right)^2 - 4\dot{r}\ddot{r}\sin\varphi\cos\varphi.\dot{\varphi} - 2r\ddot{r}\cos^2\varphi.\left(\dot{\varphi}\right)^2 - 2r\ddot{r}\sin\varphi\cos\varphi.\ddot{\varphi} + 4r\dot{r}\sin\varphi.\cos\varphi.\left(\dot{\varphi}\right)^3 + 4r\dot{r}\sin^2$$
$$\left(\ddot{r}\sin\varphi\right)^2 + \left(2\dot{r}\cos\varphi.\dot{\varphi}\right)^2 + \left[r\sin\varphi.\left(\dot{\varphi}\right)^2\right]^2 + \left(r\cos\varphi.\ddot{\varphi}\right)^2 + 4\dot{r}\ddot{r}\sin\varphi\cos\varphi.\dot{\varphi} - 2r\ddot{r}\sin^2\varphi.\left(\dot{\varphi}\right)^2 + 2r\ddot{r}\sin\varphi\cos\varphi.\ddot{\varphi} - 4r\dot{r}\sin\varphi.\cos\varphi.\left(\dot{\varphi}\right)^3 + 4r\dot{r}\cos^2$$
$$\sqrt{\left(\ddot{r}\right)^2 + \left(2\dot{r}.\dot{\varphi}\right)^2 + \left[r\left(\dot{\varphi}\right)^2\right]^2 + \left(r.\ddot{\varphi}\right)^2 - 2r\ddot{r}.\left(\dot{\varphi}\right)^2 + 4r\dot{r}.\dot{\varphi}.\ddot{\varphi}}$$

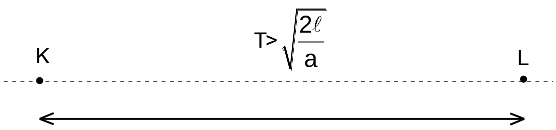
$$v = \sqrt{{v_x}^2 + {v_y}^2} = \sqrt{{v_r}^2 + {v_\varphi}^2} = \sqrt{\dot{r}^2 + r^2.\left(\dot{\varphi}\right)^2}$$
$$\vec{a}.\vec{b} = ab\cos\alpha = a_x b_x + a_y b_y$$

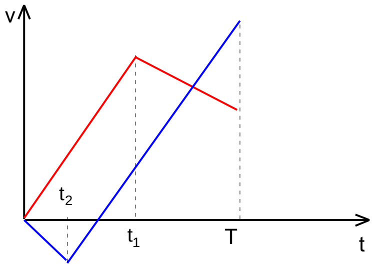
$$\vec{v}.\vec{e}_r = v_r = v_x e_{rx} + v_y e_{ry} = \left(\dot{r}\cos\varphi - r\sin\varphi.\dot{\varphi}\right)\cos\varphi + \left(\dot{r}\sin\varphi + r\cos\varphi.\dot{\varphi}\right)\sin\varphi =$$
$$= \dot{r}\cos^2\varphi - r\sin\varphi\cos\varphi.\dot{\varphi} + \dot{r}\sin^2\varphi + r\sin\varphi\cos\varphi.\dot{\varphi} = \dot{r}$$

$$\vec{v}.\vec{e}_\varphi = v_\varphi = v_x e_{\varphi x} + v_y e_{\varphi y} = -\left(\dot{r}\cos\varphi - r\sin\varphi.\dot{\varphi}\right)\sin\varphi + \left(\dot{r}\sin\varphi + r\cos\varphi.\dot{\varphi}\right)\cos\varphi =$$
$$= -\dot{r}\sin\varphi\cos\varphi + r\sin^2\varphi.\dot{\varphi} + \dot{r}\sin\varphi\cos\varphi + r\cos^2\varphi.\dot{\varphi} = r.\dot{\varphi}$$

$$e_{rx} = \cos\varphi \Rightarrow e_{ry} = \sin\varphi$$
$$e_{\varphi x} = -\sin\varphi \Rightarrow e_{\varphi y} = \cos\varphi$$
$$a_x = \ddot{x} = \ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.(\dot{\varphi})^2 - r\sin\varphi.\ddot{\varphi}$$
$$a_y = \ddot{y} = \ddot{r}\sin\varphi + 2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.(\dot{\varphi})^2 + r\cos\varphi.\ddot{\varphi}$$
$$v = \sqrt{v_x{}^2 + v_y{}^2} = \sqrt{v_r{}^2 + v_\varphi{}^2} = \sqrt{\dot{r}^2 + r^2.(\dot{\varphi})^2}$$
$$\vec{a}.\vec{b} = ab\cos\alpha = a_x b_x + a_y b_y$$
$$\vec{a}.\vec{e}_r = a_r = a_x e_{rx} + a_y e_{ry} = \left(\ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.(\dot{\varphi})^2 - r\sin\varphi.\ddot{\varphi}\right)\cos\varphi + \left(\ddot{r}\sin\varphi + 2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.(\dot{\varphi})^2 + r\cos\varphi.\ddot{\varphi}\right)\sin\varphi =$$
$$= \ddot{r}\cos^2\varphi - 2\dot{r}\sin\varphi\cos\varphi.\dot{\varphi} - r\cos^2\varphi.(\dot{\varphi})^2 - r\sin\varphi\cos\varphi.\ddot{\varphi} + \ddot{r}\sin^2\varphi + 2\dot{r}\sin\varphi\cos\varphi.\dot{\varphi} - r\sin^2\varphi.(\dot{\varphi})^2 + r\sin\varphi\cos\varphi.\ddot{\varphi} =$$
$$= \ddot{r} - r(\dot{\varphi})^2$$

$$\vec{a}.\vec{e}_\varphi = a_\varphi = a_x e_{\varphi x} + a_y e_{\varphi y} = -\left(\ddot{r}\cos\varphi - 2\dot{r}\sin\varphi.\dot{\varphi} - r\cos\varphi.(\dot{\varphi})^2 - r\sin\varphi.\ddot{\varphi}\right)\sin\varphi + \left(\ddot{r}\sin\varphi + 2\dot{r}\cos\varphi.\dot{\varphi} - r\sin\varphi.(\dot{\varphi})^2 + r\cos\varphi.\ddot{\varphi}\right)\cos\varphi =$$
$$= -\ddot{r}\sin\varphi\cos\varphi + 2\dot{r}\sin^2\varphi.\dot{\varphi} + r\sin\varphi\cos\varphi.(\dot{\varphi})^2 + r\sin^2\varphi.\ddot{\varphi} + \ddot{r}\sin\varphi\cos\varphi + 2\dot{r}\cos^2\varphi.\dot{\varphi} - r\sin\varphi\cos\varphi.(\dot{\varphi})^2 + r\cos^2\varphi.\ddot{\varphi} =$$
$$= 2\dot{r}\dot{\varphi} + r\ddot{\varphi} = \frac{1}{r}\frac{d}{dt}\left(r^2\dot{\varphi}\right) = \frac{1}{mr}\frac{d}{dt}\left(mr^2\dot{\varphi}\right) = \frac{1}{mr}\frac{dL}{dt}$$





$$x=\frac{at_1^2}{2}\Rightarrow v_1=at_1\Rightarrow \ell-x=at_1(T-t_1)-\frac{a(T-t_1)^2}{2}$$

$$\ell-\frac{at_1^2}{2}=at_1(T-t_1)-\frac{a(T-t_1)^2}{2}\Rightarrow \ell=\frac{at_1^2}{2}+aTt_1-at_1^2-\frac{aT^2}{2}+aTt_1-\frac{at_1^2}{2}=2aTt_1-at_1^2-\frac{aT^2}{2}$$

$$at_1^2-2aTt_1+\ell+\frac{aT^2}{2}=0\Rightarrow t_1=\frac{2aT-\sqrt{4a^2T^2-4a\left(\ell+\frac{aT^2}{2}\right)}}{2a}=T-\frac{\sqrt{4a^2T^2-4a\ell-2a^2T^2}}{2a}=T-\frac{\sqrt{2a^2T^2-4a\ell}}{2a}$$

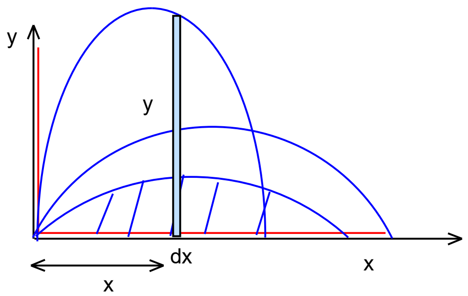
$$v_{min}=at_1-a(T-t_1)=2at_1-aT=2aT-\sqrt{2a^2T^2-4a\ell}-aT=aT-\sqrt{2a^2T^2-4a\ell}$$

$$y=\frac{at_2^2}{2}\Rightarrow v_2=at_2\Rightarrow y=\frac{at_2^2}{2}$$

$$2y+\ell=\frac{a(T-2t_2)^2}{2}\Rightarrow at_2^2+\ell=\frac{aT^2}{2}-2aTt_2+2at_2^2\Rightarrow at_2^2-2aTt_2+\frac{aT^2}{2}-\ell=0$$

$$t_2=\frac{2aT-\sqrt{4a^2T^2-4a\left(\frac{aT^2}{2}-\ell\right)}}{2a}=T-\frac{\sqrt{4a^2T^2-2a^2T^2+4a\ell}}{2a}=T-\frac{\sqrt{2a^2T^2+4a\ell}}{2a}$$

$$v_{max}=a(T-2t_2)=a\left(T-2T+\frac{\sqrt{2a^2T^2+4a\ell}}{a}\right)=\sqrt{2a^2T^2+4a\ell}-aT$$



$$x=v_0t\cos\theta\Rightarrow y=v_0t\sin\theta-\frac{gt^2}{2}\Rightarrow t=\frac{v_0\sin\theta}{g}\Rightarrow \ell=\frac{2v_0^2\sin\theta\cos\theta}{g}$$

$$y=v_0\sin\theta.\frac{x}{v_0\cos\theta}-\frac{g}{2}\left(\frac{x}{v_0\cos\theta}\right)=x\tan\theta-\frac{gx^2}{2v_0^2\cos^2\theta}=x\tan\theta-\frac{gx^2(1+\tan^2\theta)}{2v_0^2}$$

$$dS=ydx\Rightarrow S=\int_0^\ell\left[x\tan\theta-\frac{gx^2(1+\tan^2\theta)}{2v_0^2}\right]dx=\frac{x^2\tan\theta}{2}-\frac{gx^3(1+\tan^2\theta)}{6v_0^2}\Bigg|_0^\ell=$$

$$=\frac{\ell^2}{2}\left[\tan\theta-\frac{g\ell(1+\tan^2\theta)}{3v_0^2}\right]=\frac{2v_0^4\sin^2\theta\cos^2\theta}{g^2}\left[\tan\theta-\frac{2(1+\tan^2\theta)\sin\theta\cos\theta}{3}\right]=$$

$$=\frac{2v_0^4\sin^2\theta\cos^2\theta}{g^2}\left[\tan\theta-\frac{2\sin\theta}{3\cos\theta}\right]=\frac{2v_0^4\sin^2\theta\cos^2\theta}{g^2}\cdot\frac{\sin\theta}{3\cos\theta}=\frac{2v_0^4\sin^3\theta\cos\theta}{3g^2}$$

$$\frac{d}{d\theta}(\sin^3\theta\cos\theta)=3\sin^2\theta\cos^2\theta-\sin^4\theta=0\Rightarrow 3\cos^2\theta=\sin^2\theta\Rightarrow \tan\theta=\sqrt{3}$$