300 CREATIVE PHYSICS PROBLEMS

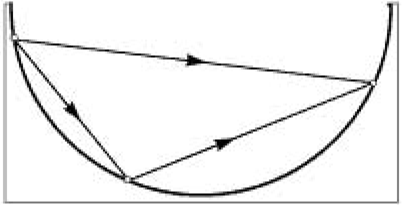
with Solutions

LÁSZLÓ HOLICS

Problem 1. A train is moving at a speed of v towards the railwayman next to the rails. The train whistles for a time of T. How long does the railwayman hear the whistle? The speed of sound is $c = 330\,\mathrm{m/s}$; $v = 108\ \mathrm{km/hour} = 30\,\mathrm{m/s}$, $T = 3\,\mathrm{s}$; the train does not reach the railwayman until the end of the whistle.

Problem 2. The speed of a motorboat in still water is four times the speed of a river. Normally, the motorboat takes one minute to cross the river to the port straight across on the other bank. One time, due to a motor problem, it was not able to run at full power, and it took four minutes to cross the river along the same path. By what factor was the speed of the boat in still water reduced? (Assume that the speed of the water is uniform throughout the whole width of the river.)

Problem 3. Consider a trough of a semicircular cross section, and an inclined plane in it that leads from a point A to point B lying lower than A. Prove that wherever point C is chosen on the arc AB, an object will always get from A to B faster along the slopes ACB than along the original slope AB. The change of direction at C does not involve a change in speed. The effects of friction are negligible.



Problem 4. The acceleration of an object is uniformly increasing, and it is $a_0 = 2\ \mathrm{m/s^2}$ at $t_0 = 0$ s and $a_1 = 3\ \mathrm{m/s^2}$ at $t_1 = 1$ s. The speed of the object at $t_0 = 0$ s is $v_0 = 1$ m/s.

a) Determine the speed of the object at $t_2 = 10$ s.

b) Determine the $v-t$ function of the motion, and then plot it in the $v-t$ coordinate system.

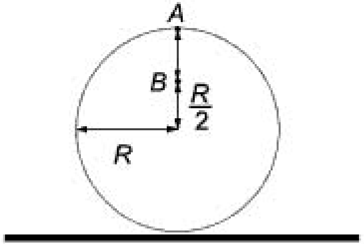
c) Estimate the distance covered by the object in the first and last second of the time interval $0 < t < 10$ s.

Problem 5. An object moves on a circular path such that its distance covered is given by the function: $s = 0.5t^2\,\mathrm{m} + 2t\,\mathrm{m}$. The ratio of the magnitudes of its accelerations at times $t_1 = 2\,\mathrm{s}$ and $t_2 = 5\,\mathrm{s}$ is $1:2$. Find the radius of the circle.

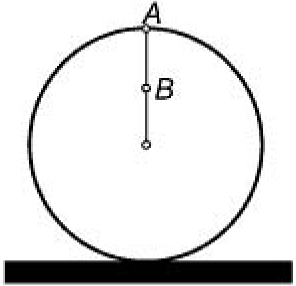
Problem 6. The radius of the tire of a car is R. The valve cap is at distance r from the axis of the wheel. The car starts from rest without skidding, at constant acceleration. Is it possible, in some way, that the valve cap has no acceleration

a) in the $\frac{1}{8}$ turn following the bottom position,

b) in the $\frac{1}{8}$ turn preceding the bottom position?



Problem 7. A disc of diameter 20 cm is rolling at a speed of 4 m/s on the ground, without slipping. How long does it take until the speed of point A first becomes equal to the present value of the speed of point B?



Problem 8. A disc of radius $R = 1$ m rolls uniformly, without skidding on horizontal ground. The speed of its centre is $v = 0.5$ m/s. Let A stand for the topmost point at $t = 0$ and B for the mid-point of the corresponding radius.

a) At what time will the speed of point A first equal the speed of point B?

b) Following on from part a) above, when the speed of point A first equals the speed of point B, what is this speed?

c) Following on from part a) above, find the distance travelled by the centre of the disk up to the time when the speed of point A first equals the speed of point B.

Problem 9. A cart moves on a muddy road. The radius of its wheels is $R = 0.6$ m. A small bit of mud detaches from the rim at a height $h = \frac{3}{2}R$ from the ground.

a) Find the speed of the cart if the bit of mud falls back on the wheel at the same height.

b) Find the length of the arc on the rim that connects the points of detaching and falling back.

c) Find the distance covered by the car in the meantime.

Problem 10. A balloon is rising vertically from the ground in such a way that with high accuracy its acceleration is a linearly decreasing function of its altitude above the ground level. At the moment of release the velocity of the balloon is zero, and its acceleration is a_0.

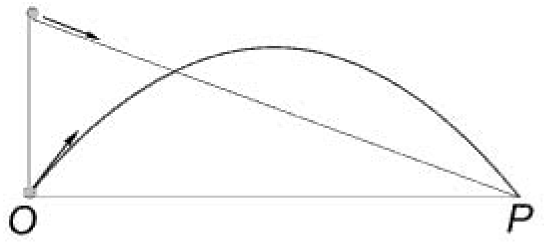
a) Determine the speed of the balloon at the height H, where its acceleration becomes zero.

b) What is the speed of the balloon at half of the altitude H?

c) How long does it take the balloon to reach the altitude H?

Problem 11. A massive ball is falling down from an initial height of $h = 20$ m. With a gun held horizontally, $d = 50$ m far from the trajectory of the falling ball, at the height of $h' = 10$ m, we are going to shoot at the falling ball. The bullet leaves the gun at a speed of $v = 100$ m/s. At what time after the start of the fall should the gun be fired in order to hit the falling ball with the bullet? (The air resistance is negligible.)

Problem 12. Two objects, one sliding down from rest on a smooth (frictionless) slope, the other being thrown from the point O, start their motion at the same instant. Both get to the point P at the same time and at the same speed. Determine the initial angle of the throw.

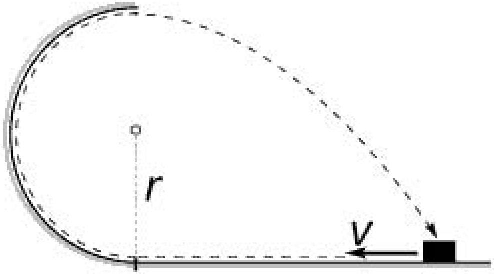


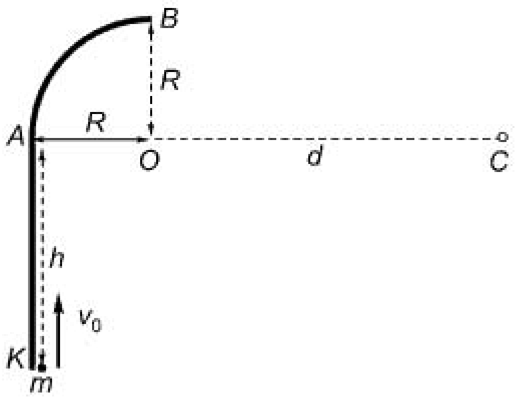
Problem 13. A projectile is projected on the level ground at an angle of 30° with an initial speed of $400\ \mathrm{m/s}$. At one point during its trajectory the projectile explodes into two pieces. The two pieces reach the ground at the same moment; one of them hits the ground at exactly where it was projected with a speed of $250\ \mathrm{m/s}$. At what height did the explosion occur? (Air drag and the mass of the explosive material is negligible, the acceleration due to gravity can be considered as $10\ \mathrm{m/s^2}$.)

Problem 14. The bullet of a poacher flying at a speed of $v = 680\ \mathrm{m/s}$ passes the gamekeeper at a distance $d = 1\ \mathrm{m}$. What was the distance of the bullet from the gamekeeper when he began to sense its shrieking sound? The speed of propagation of sound is $c = 340\ \mathrm{m/s}$.

1.2 Dynamics

Problem 15. A frictionless track consists of a horizontal part of unknown length, which connects to a vertical semicircle of radius r as shown. An object, which is given an initial velocity v, is to move along the track in such a way that after leaving the semicircle at the top it is to fall back to its initial position. What should the minimum length of the horizontal part be?





Problem 16. A pointlike object of mass m starts from point K in the figure. It slides along the full length of the smooth track of radius R, and then moves freely and travels to point C.

a) Determine the vertical initial velocity of the pointlike object.

b) What is the minimum possible distance $OC = d$, necessary for the object to slide along the entire length of the track?

c) Find the normal forces exerted by the track at points A and B.

(Let $R = 1\,\mathrm{m}$, $h = 2\,\mathrm{m}$, $d = 3\,\mathrm{m}$, $m = 0.5\,\mathrm{kg}$, use $g = 10\,\mathrm{m/s^2}$)

Problem 17. A small object starts with a speed of $v_0 = 20\,\mathrm{m/s}$ at the lowest point of a circular track of radius $R = 8.16\,\mathrm{m}$. The small object moves along the track. How big a part of the circular track can be removed, if you want to carry out the same trick? (Neglect friction, $g = 9.8\,\mathrm{m/s^2}$.)

Problem 18. A small object of mass $m = 0.5\,\mathrm{kg}$ that hangs on a string of length $L = 5.6\,\mathrm{m}$ is given a horizontal velocity of $v_0 = 14\,\mathrm{m/s}$. The string can withstand a maximum tension of $40\,\mathrm{N}$ without breaking. Where is the stone when the string breaks? Use $g = 10\,\mathrm{m/s^2}$.

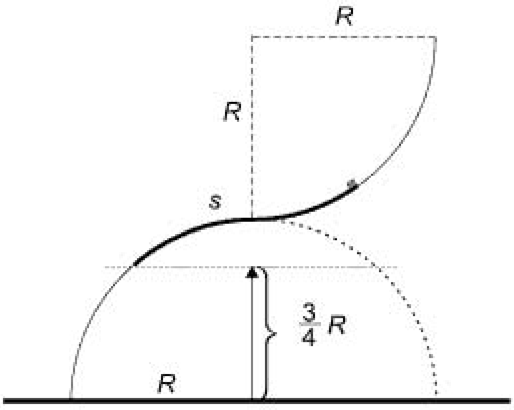
Problem 19. An object slips down the frictionless surface of a cylinder of radius R.

a) Find the position in which the acceleration of the object is two thirds of the gravitational acceleration G.

b) Find the direction of the object's acceleration in that position.

Problem 20. Two horizontal tracks are connected through two circular slopes the radii of which are equal and $R = 5\,\mathrm{m}$. The tracks and the slopes are in a vertical plane and they join without a break or sharp corner. The height difference between the horizontal tracks is $h = 2\,\mathrm{m}$. An object moves from the track at the top onto the bottom one without friction. What is the maximum initial speed of the object when it starts, in order for it to touch the path at all times during its motion?

Problem 21. A small object is moving on a special slope consisting of a concave and a convex circular arc, both of which have a right angle at the centre and radius $R = 0.5$ m, and they join smoothly, with horizontal common tangent, as it is shown in the figure. Determine the distance covered by the object on the slope, provided that it started from rest and it detaches from the slope at the altitude $\frac{3}{4}R$. (The friction is negligibly small.)

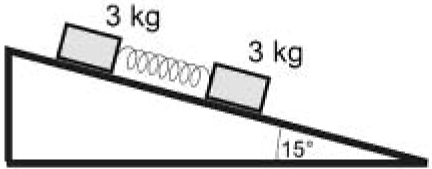


Problem 22. A pendulum, whose cord makes an angle 45° with the vertical is released. Where will the bob reach its minimum acceleration?

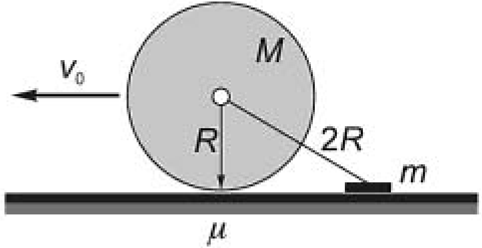
Problem 23. Two blocks, each of mass 3 kg, are connected by a spring, whose spring constant is 200 N/m. They are placed onto an inclined plane of angle 15°. The coefficient of friction between the upper block and the inclined plane is 0.3, while between the lower block and the inclined plane it is 0.1. After a while, the two blocks move together with the same acceleration. Use $g = 10\,\mathrm{m/s^2}$.

a) Find the value of their acceleration.

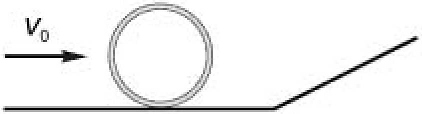
b) Find the extension of the spring.



Problem 24. A solid cylinder of mass M and radius R, rolling without sliding on a rough horizontal plane, is pulled at its axis with a horizontal velocity of v_0. By means of a string of length $2R$ attached to its axis, the cylinder is dragging a thin plate of mass $m = 2M$ lying on the plane. If the system is released, how long does it take to stop, and what is the stopping distance?
($\mu = 0.4$; $v_0 = 2$ m/s; $R = 0.5\,\mathrm{m}$, use $g = 10$ m/s^2)



Problem 25. A rigid surface consists of a rough horizontal plane and an inclined plane connecting to it without an edge. A thin hoop of radius $r = 0.1\,\mathrm{m}$ is rolling towards the slope without slipping, at a velocity of $v_0 = 3.5\,\frac{\mathrm{m}}{\mathrm{s}}$, perpendicular to the base of the slope.

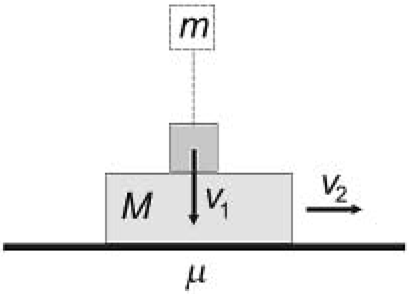


a) In which case will the hoop get higher up the slope: if there is friction on the slope or if there is not?

b) Assume that the slope is ideally smooth. At a time $t = 2.4\,\mathrm{s}$ after arriving at the slope, what will be the speed of the hoop returning from the slope?

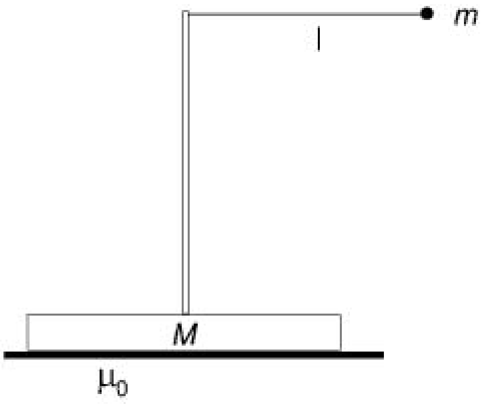
(The coefficients of both static and kinetic friction between the horizontal plane and the hoop are $\mu = 0.2$. The slope connects to the horizontal plane with a smooth curve of radius $R > r$, which is considered part of the slope. The hoop does not fall on its side during the motion.)

Problem 26. A block of mass $M = 5$ kg is moving on a horizontal plane. An object of mass $m = 1$ kg is dropped onto the block, hitting it with a vertical velocity of $v_1 = 10$ m/s. The speed of the block at the same time instant is $v_2 = 2$ m/s. The object sticks to the block. The collision is momentary. What will be the speed of the block after the collision if the coefficient of friction between the block and the horizontal plane is $\mu = 0.4$?



Problem 27. A pointlike ball of mass m is tied to the end of a string, which is attached to the top of a thin vertical rod. The rod is fixed to the middle of a block of mass M lying at rest on a horizontal plane. The pendulum is displaced to a horizontal position and released from rest.

If the coefficient of static friction between the block and the ground is μ_s, what angle will the string create with the vertical rod at the time instant when the block starts to slide? ($M = 2$ kg, $m = 1$ kg, $\mu_s = 0.2$.)

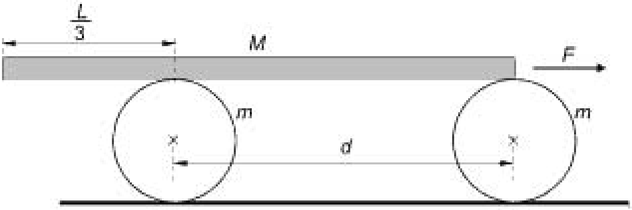


Problem 28. Two small cylinders of equal radius are rotating quickly in opposite directions. Their spindles are parallel and lie on the same horizontal plane. The distance between the spindles is 2L. Place a batten of uniform density onto the top of the two cylinders so that the batten is perpendicular to the spindles, and its centre of mass is at a distance of x from the perpendicular bisector of the segment between the two spindles, which is perpendicular to the spindles. What type of motion does the batten undergo?

Problem 29. An object is pulled up uniformly along an inclined plane which makes an angle of α with the horizontal. The angle between the force with which it is pulled up and the plane of the incline is β. The coefficient of friction between the plane and the object is μ. In what interval can the angle β vary to allow the force to pull up the object?

Problem 30. A coin is placed onto a phonograph turntable at a distance of $r = 10$ cm from the centre. The coefficient of static friction between the coin and the turntable is $\mu = 0.05$. The turntable, which is initially at rest, starts to rotate with a constant angular acceleration of $\beta = 2\ \mathrm{s}^{-2}$. How much time elapses before the coin slips on the turntable?

Problem 31. A rigid rod of length $L = 3$ m and mass $M = 3$ kg, whose mass is distributed uniformly, is placed on two identical thin-walled cylinders resting on a horizontal table. The axes of the two cylinders are $d = 2$ m from each other. As for the rod, one of its endpoints is directly above the axis of one cylinder, while its trisector point (closer to its other end) is directly above the axis of the other cylinder. The mass of the cylinders is $m = 1$ kg each. A constant horizontal pulling force $F = 12$ N acts on the rod. Both cylinders roll without friction.

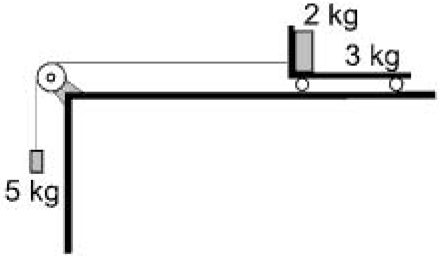


a) Find the final speed of the rod, when its leftmost end is exactly above the axis of the left cylinder.

b) Find the friction force and the minimum coefficient of friction required between the cylinders and the rod for pure rolling.

c) Find the minimum coefficient of friction between the table and the cylinders.

Problem 32. A cart of mass 3 kg is pulled by a 5 kg object as shown. The cart, whose length is 40 cm moves along the table without friction. There is a brick of mass 2 kg on the cart, which falls from it 0.8 s after the start of the motion. Find the coefficient of kinetic friction between the cart and the brick. Use $g = 10\,\mathrm{m/s^2}$.



Problem 33. A small solid sphere of mass $m = 8\,\mathrm{kg}$ is placed inside a rigid hollow sphere of mass $M = 8\,\mathrm{kg}$. The hollow sphere is then dropped from a great height. Air drag is in direct proportion to the square velocity: $F = kv^2$. If speed and force are measured in m/s and Newton respectively, then $k = 0.1$. Draw a graph that represents the force exerted by the small sphere on the hollow sphere in terms of velocity. Use $g = 10\,\mathrm{m/s^2}$.

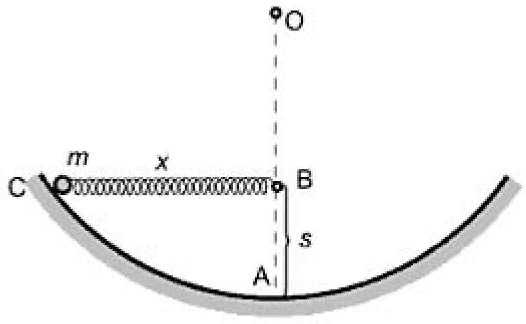
Problem 34. A small body that is fixed to the end of a string of length $l = 20\,\text{cm}$ is forced to move along a circle on a slope whose angle of inclination is $\alpha = 30°$. The body starts from the lowest position in such a way that its speed at the topmost position is $v = 3\,\text{m/s}$.

a) Find the initial velocity, if at the topmost point, the tension in the string is half of what it is at the moment of starting.

b) Find the coefficient of friction.

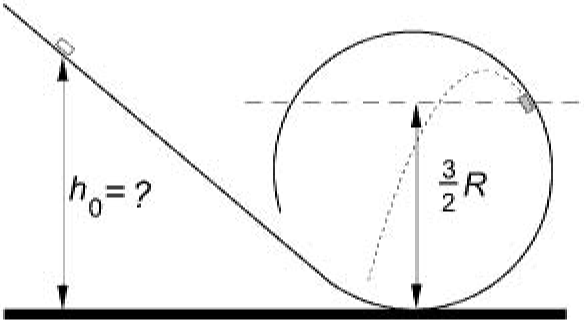
c) Find the distance travelled by the body until stopping, if after $5/4$ turns the string is released and the body remains on the slope throughout its motion.

Problem 35. The inner radius of a frictionless spherical shell is $OA = 0.8\,\text{m}$. One end of a spring of relaxed length $L = 0.32\,\text{m}$ and spring constant $D = 75\,\text{N/m}$ is fixed to point B, which is $0.48\,\text{m}$ below the centre of the sphere. A ball of mass $m = 3.2\,\text{kg}$ is attached to the other end of the spring, while the spring is extended in a horizontal position to reach point C. Then the ball is released. ($g = 10\,\text{m/s}^2$)



a) Find the speed of the ball when it has traveled furthest down the cylinder.

b) Find the force exerted by the ball on the spherical shell at that point.



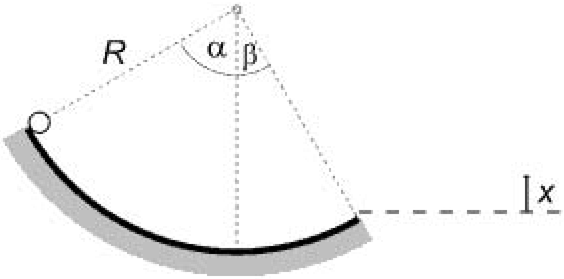
Problem 36. A tangentially attached slope leads to a circular match-box track with radius $R = 32$ cm set in a vertical plane. The toy car starts from rest at the top of the slope, runs down the slope and detaches from the track at height $h = \dfrac{3}{2}R$ measured from the bottom.

a) Find the height the car starts from.

b) Find the maximum height reached by it after it reaches the bottom of the track.

(Assume that the toy car is point-like, neglect drag and friction.)

Problem 37. A small wheel, initially at rest, rolls down a ramp in the shape of a quarter circle without slipping. The radius of the circle is $R = 1$ m and $\alpha = 60°$, $\beta = 30°$. Find the height x reached by the wheel after leaving the track.

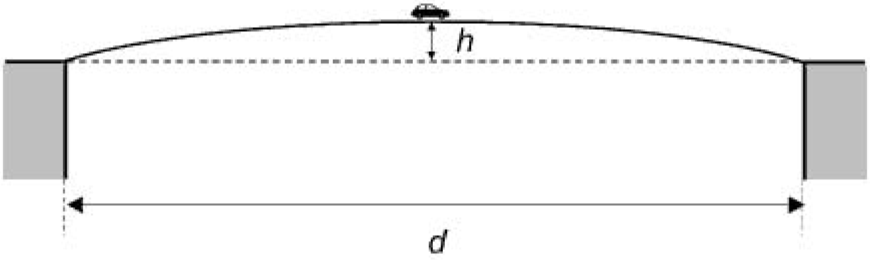


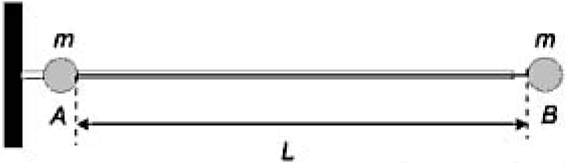
Problem 38. Two banks of a river whose width is $d=100\,\mathrm{m}$ are connected by a bridge whose longitudinal section is a parabola arc. The highest point of the path is $h=5\,\mathrm{m}$ above the level of the banks (see the figure). A car with mass $m=1000\,\mathrm{kg}$ traverses the bridge at a constant speed of $v=20\,\mathrm{m/s}$. Find the magnitude of the force that the car exerts on the bridge

a) at the highest point of the bridge,

b) at 3/4 of the distance between the two banks.

(Drag can be neglected. Calculate with $g=10\,\mathrm{m/s^2}$.)





Problem 39. An iron ball (A) of mass $m = 2$ kg can slide without friction on a fixed horizontal rod, which is led through a diametric hole across the ball. There is another ball (B) of the same mass m attached to the first ball by a thin thread of length $L = 1.6$ m. Initially the balls are at rest, the thread is horizontally stretched to its total length and coincides with the rod, as is shown in the figure. Then the ball B is released with zero initial velocity.

a) Determine the velocity and acceleration of the balls (A) and (B) at the time when the thread is vertical.

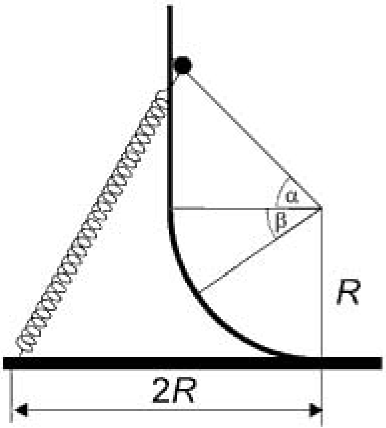
b) Determine the force exerted by the rod on the ball (A) and the tension in the thread at this instant. (In the calculations take the gravitational acceleration to be $g = 10\ \mathrm{m/s^2}$.)

Problem 40. A plane inclined at an angle of 30° ends in a circular loop of radius $R = 2\ \mathrm{m}$. The plane and the loop join smoothly. A marble of radius $r = 1\ \mathrm{cm}$ and of mass $m = 20\ \mathrm{g}$ is released from the slope at a height of $h = 3R$. What is the lowest value of the coefficient of friction if the marble rolls along the path without sliding?

Problem 41. The vertical and horizontal parts of a track are connected by a quarter of a circular arc whose radius is $R = 0.2$ m. A ball slides on the track with negligible friction; it is pulled through a slit along the track by a stretched spring as is shown in the figure. The length of the unstretched spring is 0.2 m, the spring constant is 100 N/m. The sliding ball starts from a point that is higher than $\alpha = 45^\circ$ above the horizontal part of the track when viewed from the centre of the arc and reaches the maximum velocity at angle $\beta = 34^\circ$ below the horizontal part of the track.

a) Find the mass of the ball.

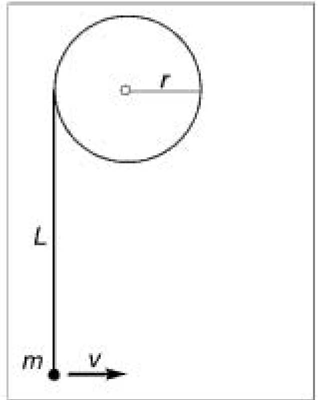
b) Find the maximum speed of the ball.



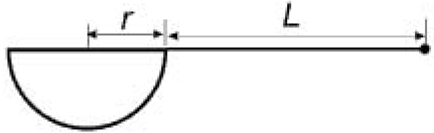
Problem 42. A horizontal disk of radius $r = 0.2\,\mathrm{m}$ is fixed onto a horizontal frictionless table. One end of a massless string of length $L = 0.8\,\mathrm{m}$ is fixed to the perimeter of the disk, while the other end is attached to an object of mass $m = 0.6\,\mathrm{kg}$, which stands on the table as shown. The object is then given a velocity of magnitude $v = 0.4\,\mathrm{m/s}$ in a direction perpendicular to the string.

a) At what time will the object hit the disk?

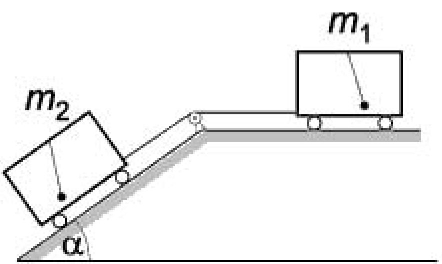
b) Find the tension in the string as a function of time.



Problem 43. A semi-cylinder of radius $r = 0.5$ metres is fixed in horizontal position. A string of length L is attached to its edge. The object tied to the end of the string is released from a horizontal position. When the object at the end of the string is rising, at a certain point the string becomes slack. When the string becomes slack, the length of the free part of the string is $s = 0.96r = 0.48$ metres. What is the total length of the string?



Problem 44. On a horizontal table with the height $h = 1\,\mathrm{m}$ there is a block of mass $m_1 = 4\,\mathrm{kg}$ at rest. The block is connected by a long massless string to a second block of mass $m_2 = 1\,\mathrm{kg}$ which hangs from the edge of the table. The blocks are then released. Find the distance between the points where the two blocks hit the ground. Neglect friction.

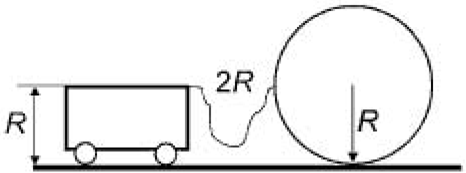


Problem 45. Two carts of masses $m_1 = 8\,\mathrm{kg}$ and $m_2 = 17\,\mathrm{kg}$ are connected by a cord that passes over a pulley as shown. Cart m_2 stands on an incline with an angle $\alpha = 36°52'$. If the system is released, what would be the positions of the pendulums inside the two carts? Neglect friction.

Problem 46. A solid cube of mass $m = 8\,\mathrm{kg}$ and edge $l = 20\,\mathrm{cm}$ is lying at rest on a smooth horizontal plane. A string of length l is attached to the midpoint of one of its base edges.

With the other end of the string kept on the plane, the cube is pulled with the string at an acceleration of $a = 3g$. The string stays perpendicular to the edge of the cube that it pulls on. Find the constant force exerted by the cube on the ground and the force exerted by the string on the cube.

Problem 47. A uniform solid disc of radius R and mass m is pulled by a cart on a horizontal plane with a string of length $2R$ attached to its perimeter. The other end of the string is attached to the cart at a height R above the ground. In the case of equilibrium, what angle does the string create with the horizontal plane if

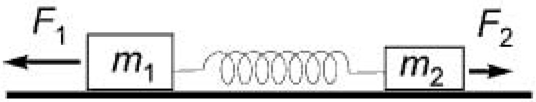


a) there is no friction,

b) there is friction?

The axis of the disc is perpendicular to both the string and the velocity.

Problem 48. The system shown in the figure undergoes uniformly accelerated motion. Data: $m_1 = 10$ kg, $F_1 = 20$ N, $m_2 = 2$ kg, $F_2 = 10$ N. Find the reading on the spring scale:

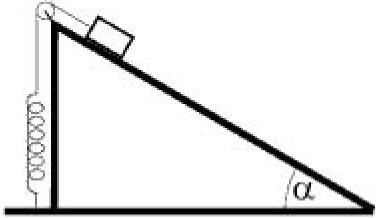


a) in this arrangement,

b) if the forces F_1 and F_2 are swapped,

c) if $m_1 = m_2 = 6$ kg. How does the result change in cases a) and b) if m_2 is negligibly small in comparison to m_1, for example $m_2 = 10$ g? Friction is negligible and the mass of the spring is negligible as well.

Problem 49. A block of mass $m=3\,\mathrm{kg}$ is connected to a spring and held on top of an inclined plane of angle $\alpha = 30^\circ$ as shown. The spring, whose spring constant is $D = 80\,\mathrm{N/m}$ is in its relaxed state when the block is released. The coefficient of friction is very small. Use $g=10\,\mathrm{m/s^2}$.

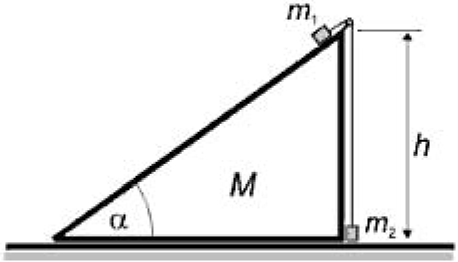


a) What is the greatest depth reached by the block?

b) Where will the very small friction make the block stop?

Problem 50. A body of mass m is placed on a wedge whose angle of inclination is α and whose mass is M. Find the horizontal force F that should be applied on the wedge in order for the body of mass to slide from the top to the bottom of the wedge in twice as much time as it would if the wedge were stationary. The friction between the wedge and the horizontal ground can be neglected, the coefficient of friction between the wedge and the body is μ. Initially both bodies are at rest. ($M=1\,\mathrm{kg}$, $m=1\,\mathrm{kg}$, $\alpha=30^\circ$, $\mu=0.2$, $g=9.81\,\mathrm{m/s^2}$)

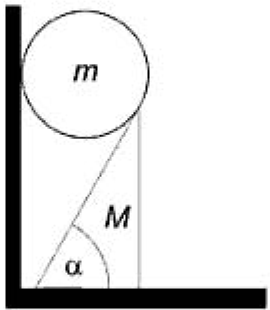
Problem 51. A block of mass $m_1 = 7$ kg is placed on top of a $h = 1$ m high inclined plane with an angle $\alpha = 36.87°$ and mass $M = 2$ kg which is connected by a cord of length h over a massless, frictionless pulley to a second block of mass $m_2 = 1$ kg hanging vertically as shown. The inclined plane can move without friction in the horizontal direction. The blocks are then released. After how long will the two blocks be nearest to each other? Neglect friction and use $g = 10\,\mathrm{m/s^2}$.



Problem 52. A sphere of mass m is placed between a vertical wall and a wedge of mass M and angle α, in such a way that the sphere touches the wedge tangentially at the topmost point of the wedge, as is shown in the figure. The wedge is standing on a horizontal plane, and both the sphere and the wedge move without friction.

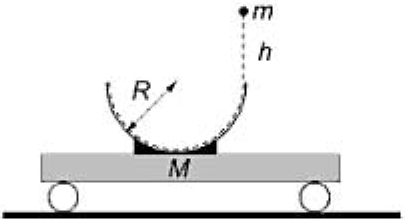
a) How should the mass ratio M/m and the angle α be chosen so that the wedge does not tilt after releasing the sphere?

b) Determine the speed reached by the sphere by the time it slides along a segment of length $l = 20$ cm of the wedge, provided that $\alpha = 60°$ and $M/m = 12$.



Problem 53. A large, closed box slides down on a very long, inclined plane. An observer inside the box wants to determine the angle of the inclined plane (α) and the coefficient of kinetic friction. What experiments should he do and what should he measure in order to be able to calculate the above quantities?

Problem 54. A thin, rigid wooden rod of height h is fixed to the ground and is standing vertically. A simple pendulum of length $l < h$ and mass m is attached to its upper end. The pendulum is moved to a horizontal position and released. Determine the torque that the fixed lower end must bear to keep the rod in position. (Let $h = 1.2\,\text{m}$, $m = 0.5\,\text{kg}$, use $g = 10\,\text{m/s}^2$)

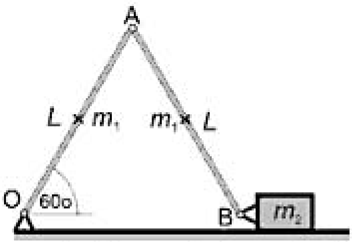


Problem 55. As shown in the figure, a smooth hemisphere of radius R is fixed to the top of a cart that can roll smoothly on a horizontal ground. The total mass of the cart is M, and it is initially at rest. A pointlike ball of mass m is dropped into the hemisphere tangentially, from a point $h = R$ above its edge. The ball slides all the way along the hemisphere with negligible friction.

a) Where will the ball be when it reaches the maximum height during its motion?

b) With what force will the ball press on the hemisphere at its lowermost point?

(Let $R = 0.5\,\mathrm{m}$, $M = 2\,\mathrm{kg}$, $m = 0.5\,\mathrm{kg}$, use $g = 10\,\mathrm{m/s^2}$)

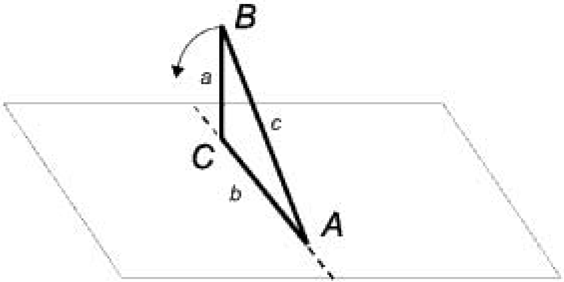


Problem 56. Two rods, each of length $L = 0.5\,\mathrm{m}$ and mass $m_1 = 1\,\mathrm{kg}$, are joined together by hinges as shown. The bottom end of the left rod is connected to the ground, while the bottom end of the right one is connected to a block of mass $m_2 = 2\,\mathrm{kg}$. The block is then released to a position where the rods form a 60° angle with the horizontal plane. Friction is negligible.

a) Find the velocity of point A as it hits the ground.

b) Find the acceleration of mass m_2 at that moment.

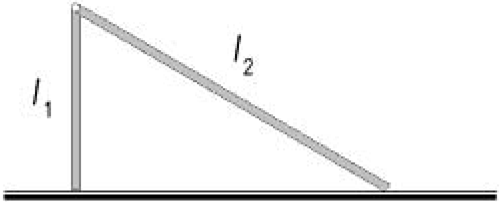
Problem 57. A right triangle of side lengths a, b and c is formed using three thin rods of the same material, which are firmly fixed to each other. The triangle, which is initially placed vertically onto a horizontal plane on its edge b, tumbles down from this unstable position. $a=30$ cm, $c=50$ cm.



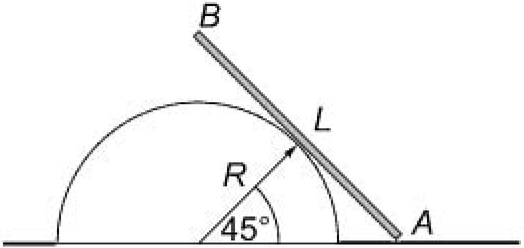
a) Determine the velocity of the vertex B when it hits the horizontal plane, provided that the triangle does not slide along ground.

b) Determine the position and velocity of the vertex B when it hits the horizontal plane, provided that the friction is negligible!

Problem 58. Two thin rods of identical material and cross-section with lengths $l_1 = 0.6$ m and $l_2 = 1$ m are connected by a frictionless joint. The structure slides from its unstable equilibrium position in such a way that the rods remain on a vertical plane and the angle enclosed by them decreases. Find the place where the joint reaches the ground and find its speed upon impact.

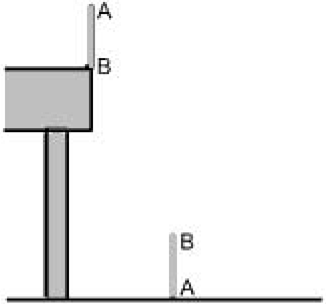


Problem 59. As shown in the figure, a thin and solid rod of length $L = 2R$ is leaning against a smooth semi-cylinder of radius $R = 1$ m that is fixed to a horizontal plane. The lower end of the rod A is held on the ground and then released from rest. The rod falls, sliding along the side of the cylinder. What will be the speed of its upper end B at the time instant when it reaches the surface of the cylinder? (Neglect all friction.)



the surface of the cylinder? (Neglect all friction.)

Problem 60. A rod with length L stands on the edge of a table in such a way that its bottom is propped against a smooth (frictionless) peg. Then the rod tilts and falls. Find the height of the table if the rod reaches the floor in a vertical position, with its top end hitting the floor first.



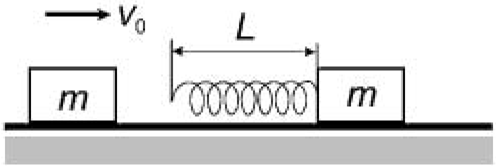
Problem 61. The following forces act on a body, which is initially at rest: $F_1 = 10$ N for $t_1 = 4$ s, then $F_2 = 4$ N acting in the same direction for $t_2 = 14$ s, then $F_3 = 15$ N acting in the opposite direction for $t_3 = 2$ s.

Find the magnitude of the constant force that causes the same final velocity of the body:

a) at the same time,

b) at the same distance.

Problem 62. A block of mass m with a spring fastened to it rests on a horizontal frictionless surface. The spring constant is D_0, the relaxed length of the spring is L and the spring's mass is negligible. A second block of mass m moves along the line of axis of the spring with constant velocity v_0 and collides with the spring as shown.



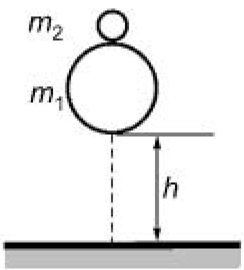
a) What is the shortest length of the spring during the collision?

b) The second block then sticks to the left end of the spring. What is the frequency of oscillation of the system?

Values: $m = 1\,\text{kg}$, $L = 0.2\,\text{m}$, $D_0 = 250\,\text{N/m}$, $v_0 = 0.8\,\text{m/s}$.

Problem 63. Our model rocket is a trolley on which several spring launchers are installed. Each spring is compressed and therefore stores $E = 100$ J of elastic energy. The system, whose total mass is $M = 100$ kg is initially at rest. Find the velocity of the trolley if the structure shoots out three balls with mass $m = 5$ kg in succession and in the same direction along the longitudinal axis.

Problem 64. A ball of mass m and of speed v collides with a stationary ball of mass M. The collision was head-on but not totally elastic. Determine the kinetic energy which is lost during the collision as a function of the speeds as well as the given masses before and after the collision. Based on the result, define a quantity which characterizes the elasticity of the collision.

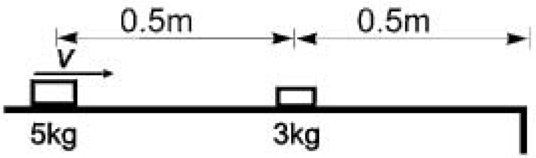


Problem 65. An object of mass m_1 and another of mass m_2 are dropped from a height h, the second one immediately following the first one. All collisions are perfectly elastic and occur along a vertical line.

a) For what ratio of the masses will the object of mass m_1 remain at rest after the collisions?

b) According to a), how high will the object of mass m_2 rise?

Problem 66. Two blocks of masses $m_1 = 5\,\text{kg}$ and $m_2 = 3\,\text{kg}$ are at rest on a table at a distance of $s_1 = 0.5\,\text{m}$ from each other. Block m_2 is at a distance of $s_2 = 0.5\,\text{m}$ from the edge of the table as shown. The coefficient of friction is $\mu = 0.102 = 1/9.8$.

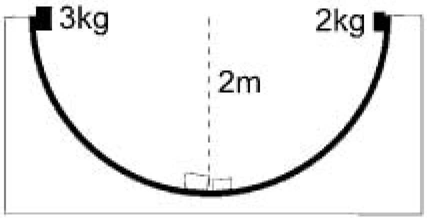


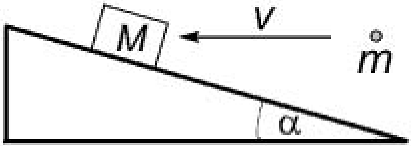
Find the velocity that should be given to block m_1 if after the elastic collision of the two blocks

a) block m_1,

b) block m_2 is to reach the edge of the table and stop there.

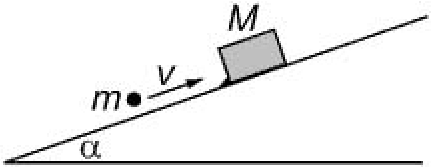
Problem 67. At the rim of a hollow hemisphere of diameter 4 metres two objects of masses $m_1 = 3\,\text{kg}$ and $m_2 = 2\,\text{kg}$ are released at the same moment. Initially the two objects are at the two endpoints of a diameter of the hemisphere. They collide totally elastically. After the first collision what are the greatest heights the blocks can reach? The friction is negligible.





Problem 68. A block of mass $M = 1.6\,\mathrm{kg}$ is lying on a plane inclined at an angle of $\alpha = 16.25^\circ$ to the horizontal. The coefficient of friction is $\mu = 0.2$. At the same time that the block on the inclined plane is released, a shell of mass $m = 0.4\,\mathrm{kg}$ is fired into it horizontally with a speed of $v = 12\,\mathrm{m/s}$. How much will the block of mass and the shell of mass slide up the incline? ($g = 10\,\mathrm{m/s^2}$)

Problem 69. A block of mass M, supported by a buffer, stays at rest on a plane inclined at an angle α to the horizontal. From below, parallel to the inclined plane, a bullet of mass m is shot into the block at a speed of v. How long does it take for the block to reach the buffer again? The coefficient of friction between the block and the plane is μ. The bullet penetrates into the block. During the penetration the displacement of the block is negligible. The coefficients of static and kinetic friction can be considered equal.



Problem 70. A ball made of a totally inelastic material is hung between two heavy iron rods, which are also hung as pendulums. The mass of the ball is negligible with respect to that of the rods. The masses of the rods are: m_1 and m_2, $(m_1 > m_2)$. One of the rods is pulled out, so that its centre of mass rises to a height of h, and then it is released. The plastically deformable ball becomes flat due to the collision. Which rod should be raised in order to cause the greater compression of the ball if h is the same in both cases. Based on the result, draw a conclusion about the efficiency of deforming an object by hammering it.

Problem 71. A projectile thrown upwards explodes at the top of its path into two parts of masses $m_1 = 3$ kg and $m_2 = 6$ kg. The two parts reached the ground at equal distances from the position of the projection, and with a time difference of $T = 4$ seconds. At what height did the projectile explode? (Neglect air resistance.)

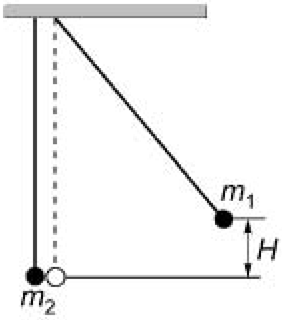
Problem 72. From a horizontal ground a projectile is shot at an initial speed of $v_0 = 150$ m/s and at an angle of $\alpha = 60°$ from the horizontal ground. After a time of $t_1 = 10$ s the projectile explodes and breaks up into two pieces of masses m and $2m$. At the moment $\Delta t = 10$ s after the explosion the piece of mass m hits the ground at a distance of $d = 500$ m behind the place of shooting, in the plane of the trajectory of the unexploded projectile. At this instant how far is the other piece of mass $2m$ from the cannon?

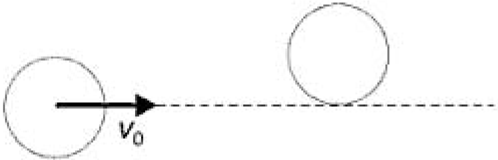
Problem 73. A trolley of mass $M = 20$ kg is travelling at a speed of $V = 10$ m/s. A spring, initially compressed, launches an object of mass $m = 2$ kg off the trolley in a forward direction in such a way that after the launch the speed of the object is $v = 2$ m/s relative to the trolley. Determine the kinetic energy of the object relative to the ground.

Problem 74. Two elastic balls are suspended at the same height; one has mass $m_1 = 0.2\,\mathrm{kg}$, the other has mass m_2. If the system is left alone in the position shown in the figure, we find that – after an elastic and central collision – both balls rise to the same height.

a) Find the mass of the other ball.

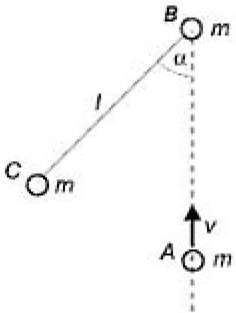
b) At what fraction is height h reached by the balls after the collision of H?

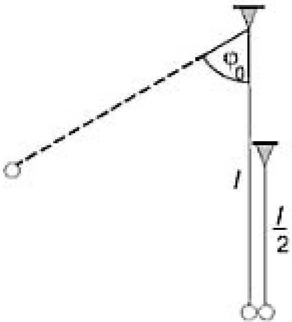




Problem 75. There are two thin, homogeneous disks of the same radius and mass lying on a horizontal air cushion table. One of the disks is at rest, while the other is moving at a speed of $v_0 = 1$ m/s. The line going through the centre of the moving disk, which follows the direction of its velocity, touches the other disk tangentially. The two disks collide elastically. Determine the velocities of the disks after the collision. The directions of the velocities can be described by angles relative to the initial velocity $\vec{v}_0$. In the process investigated friction is negligible everwhere.

Problem 76. There are three thin disks of identical mass ($m_A = m_B = m_C = m$) and radius lying at rest on a smooth horizontal plane. The disks B and C are connected by a thin thread of length $l = 1$ m. Initially the thread is straight, but not stretched, and it makes an angle of $45°$ with the line going through the midpoints of the disks A and B. Now we push the disk A at a speed $v = 2$ m/s in such a way that it centrally collides with the disk B. The collisions are elastic and instantaneous. At what time after the collision of the disks A and B will the line connecting the centres of the disks B and C be parallel to the trajectory of the disk A? At this instance, determine the distance of the disk A from B and C. (The disks can be considered pointlike.)

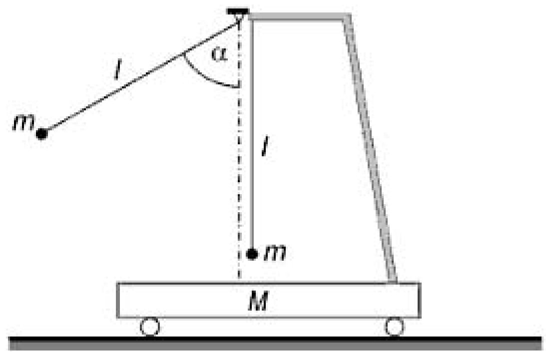




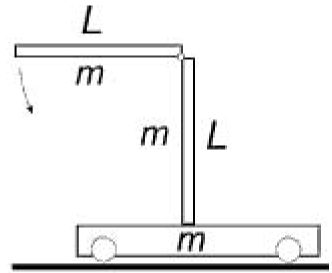
Problem 77. There are two identical balls of mass $m = 0.2$ kg suspended on two threads of lengths $l = 1$ m and $l/2$. The threads are made of the same material, and in their vertical position the two balls touch each other. If the ball hanging on the longer thread is released from an initial angle of $\varphi_0 = 60^\circ$ with respect to the vertical, then the thread breaks just before the collision.

What is the maximum initial angle from which this ball can be released, so that none of the threads break after the totally elastic collision?

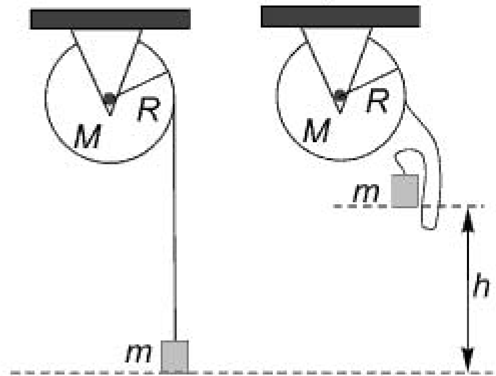
Problem 78. A mathematical pendulum of length l and mass m is suspended on a smoothly running trolley of mass M. Another pendulum, also of length l and mass m is suspended from the ceiling, displaced through angle α and then released without initial velocity. The two pendulums collide centrally and perfectly elastically. Find the angle φ through which the pendulum suspended from the trolley swings out.
($M = 3$ kg, $m = 2$ kg, $\alpha = 60^\circ$.)



Problem 79. A small cart of mass m is at rest on a horizontal track. A vertical column of length $L = 2\,\text{m}$ and the same mass m is fixed to the cart. A rod of the same mass m and length $L = 2\,\text{m}$ is attached to its upper end with a hinge, and released from a horizontal position. At what speed will the end of the rod hit the base of the column? ($g = 10\,\text{m/s}^2$.)



Problem 80. A cylinder of mass M and radius R can rotate freely about its horizontal axis. A thread is wound around its lateral surface, and a weight of mass m is attached to the free end of the thread. Initially the thread below the cylinder is vertical, and unstrained. Then the weight is lifted to a height h, and released from that position at zero initial speed. At what time after its release does the weight cover the distance $2h$?

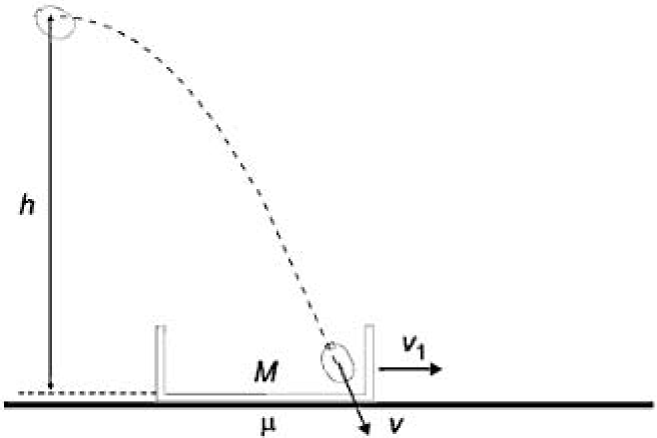


(The thread is unstretchable, and the interaction is instantaneous and totally inelastic.

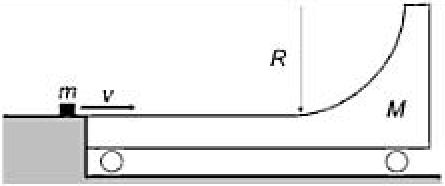
Data: $M = 2$ kg, $R = 0.2$ m, $m = 3$ kg, $h = 1.2$ m.)

Problem 81. An inclined plane of angle α and mass M can move on the ground without friction. A small object of mass m and vertical velocity v collides with the stationary inclined plane. Assuming that the collision is elastic, find the velocity of the object (u) after the collision, the angle (φ) formed by this velocity and the horizontal ground. Find the speed (c) of the inclined plane after the collision. Data: $\alpha = 36.87°$, $m = 6\,\text{kg}$, $M = 18\,\text{kg}$, $v = 14\,\text{m/s}$.

Problem 82. A big chest of mass $M = 50$ kg is sliding on the horizontal ground, and a sand bag of the same mass is falling into it. The bag was projected from the initial height of $h = 3$ m at certain a horizontal speed, and when it hits the chest, the speed of the chest is $v_1 = 5\,\text{m/s}$. The velocity of the chest is in the plane of the trajectory of the bag, and the bag hits the chest in such a way,



that its velocity makes an angle of $60°$ with the velocity of the chest. The coefficient of kinetic friction between the chest and the ground is $\mu = 0.4$. The collision of the bag is instantaneous. Determine the distance covered by the chest from the collision until it stops. What would the distance be if the bag was not thrown into the chest?



Problem 83. A quarter-round slope of radius $R = 0.5$ m is attached tangentially to a freely rolling trolley of mass $M = 3$ kg, originally at rest. A small-sized body of mass $m = 2$ kg slides onto the trolley at velocity $v = 15$ m/s.

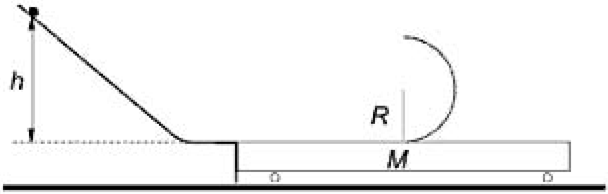
a) Find the velocity of the trolley when the small body leaves it.

b) Find the distance travelled by the trolley from parting to reunion with the body.

c) Find the velocities of the trolley and the body when they part from each other again. (Friction and drag can be neglected. Calculate with $g = 10\,\mathrm{m/s^2}$.)

Problem 84. A small object of mass $m = 1$ kg is released from rest at the top of an inclined plane that connects to a horizontal plane without an edge. It slides onto a cart of mass M that has a semi-cylindrical surface of radius $R = 0.36$ m fixed to the middle of it, as shown in the figure.

The small object reaches the topmost point of the semi-cylinder and stops there. In continues to move vertically with free fall and hits the cart exactly at the edge. All friction can be neglected.

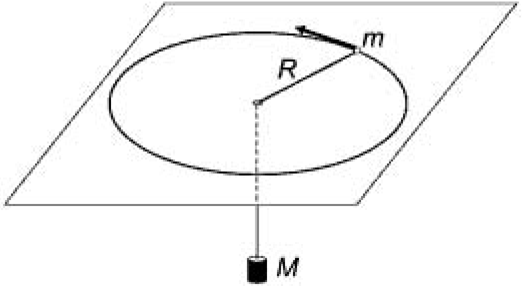


a) What is the minimum possible length of the cart?

b) What is the mass of the cart?

c) At what height h was the small object released?

Problem 85. An 80 kg man stands on the rim of a 300 kg rotating disk with radius 5 m. The disk initially rotates at $0.1\ s^{-1}$ around a vertical axis. Then the man walks from the rim to the centre of the disk. Find the change in the energy of the system.



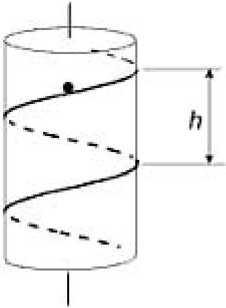
Problem 86. An object of mass $m = 1$ kg attached to a string is moving in a circle of radius $R = 40$ cm on a horizontal surface. The other end of the string is threaded through a hole at the centre of the circle and a mass of $M = 2$ kg is hung from it.

If the mass M is released, the closest approach of the mass m to the centre will be $r = 10$ cm.

a) Find the smallest and largest speeds of the mass m.

b) What is the speed of each object when the mass m is at a distance of $R/2$ from the centre?

c) Find the accelerations of the mass M at the highest and lowest points. (Neglect all friction, use $g = 10\,m/s^2$.)

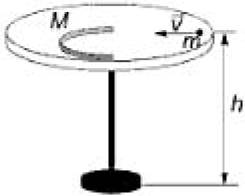


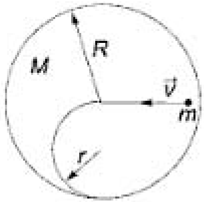
Problem 87. A solid cylinder of radius $R = 0.2$ metres is supported at the endpoints of its axis by frictionless pin bearings. An object slides down a frictionless helical track threaded around the cylindrical surface. The mass of the object is one fifth of the mass of the cylinder. The pitch of the track is $h = 0.2$ metres. $g = 10\,\mathrm{m/s^2}$.

a) What will be the speed of the object when it has descended through a height of $h = 0.2$ metres below the starting point?

b) How long will it take to attain that speed?

Problem 88. A disc of mass 2 kg and radius $R = 0.5\,\mathrm{m}$ can rotate freely around a vertical axis supported by bearings at a height $h = 1\,\mathrm{m}$ from the ground. A constraining vertical surface of negligible mass, whose shape is a semicircular arc of radius $r = R/2$, is fixed on the disc as shown in the figure. A small ball of mass $m = 1\,\mathrm{kg}$ is placed on the stationary disc and is bowled at a speed $v = 3\,\mathrm{m/s}$ in such a way that it reaches the internal side of the constraining surface tangentially.



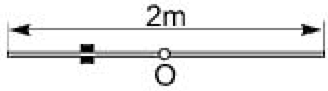


a) Find the distance from the rim of the disc where the ball reaches the ground.

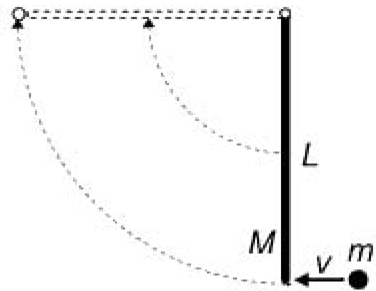
b) How far is the ball at the moment of reaching the ground from the point of leaving on the disc? (Every type of friction can be neglected.)

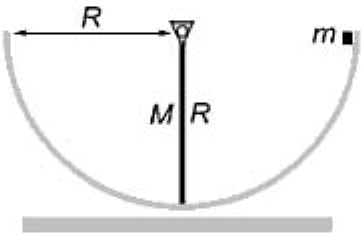
Problem 89. A pointmass moves on the frictionless inner surface of a spherical shell, whose inner radius is $R = 1.4\,\mathrm{m}$. Its velocity reaches its maximum and minimum at heights $h_1 = 0.1\,\mathrm{m}$ and $h_2 = 0.3\,\mathrm{m}$ respectively. Find the maximum and minimum values of the velocity.

Problem 90. A $2\,\mathrm{m}$ long rod of negligible mass is free to rotate about its centre. An object of mass $3\,\mathrm{kg}$ is threaded into the rod at a distance of $0.5\,\mathrm{m}$ from its end in such a way that the object can move on the rod without friction. The rod is then released from its horizontal position. Find the speed of the rod's end in the rod's vertical position. Use $g = 10\,\mathrm{m/s^2}$.



Problem 91. A board of length $L = 3.06\,\mathrm{m}$ and mass $M = 12\,\mathrm{kg}$ hangs vertically on a hinge that is connected to one of its ends. A bullet of mass $m = 0.25\,\mathrm{kg}$ is fired into the bottom end of the board, making the board swing up. What should the velocity of the bullet be if the board is to swing up to the horizontal position?

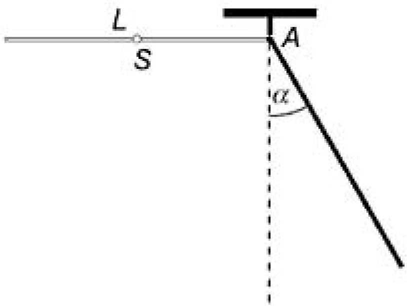




Problem 92. A rod of mass M and length R is fixed to a horizontal axis of rotation above a track with a semicircular cross section and a radius R, as is shown in the figure.

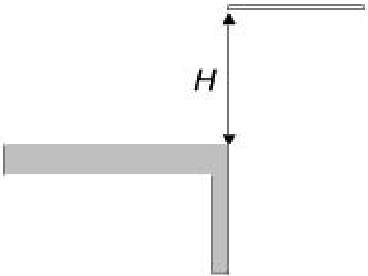
a) Find the mass of the rod relative to the mass of a point-like body that starts on the track at a height of R if it stops after an elastic collision with the rod.

b) Find the angular displacement of the rod after the collision. (Friction is negligible everywhere.)



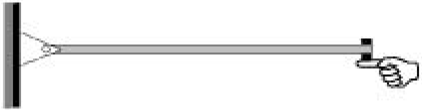
Problem 93. A thin, homogeneous stick has a length $L = 1$ m. An axis perpendicular to this stick is fixed at one end (A) of the stick; the stick is hung on the axis by a hook. The stick is sent into a horizontal position and then released without an initial velocity. The hook forms a (small) arc which allows the stick to leave it when it encloses an angle of $\alpha = 30^\circ$ with the vertical plane.

Find the angle enclosed by the stick and the horizontal at the moment when its centre of mass (S) is at the highest point after detachment from the hook.



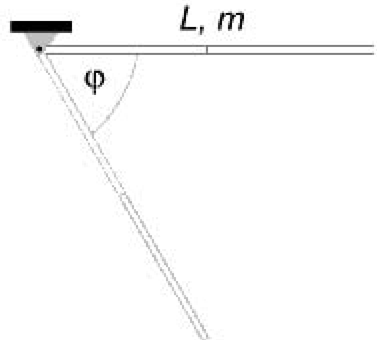
Problem 94. A thin rod of length L is falling freely in horizontal position from a height H above the surface of the table, in such a way that the end of the rod just hits the edge of the table. This collision is instantaneous and totally elastic. At what time after the collision does the rod perform a whole revolution? Where is its centre at that moment?

($H = 80$ cm, $L = 40$ cm, calculate with free fall acceleration $g = 10$ m/s.)

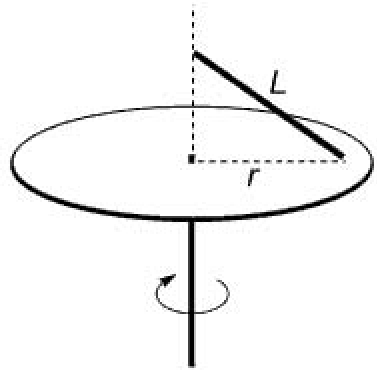


Problem 95. One end of a thin and heavy rod of length $L = 1$ m is attached to a horizontal axis at a height of $2L$ above the ground, and the rod is held in a horizontal position. One of two pointlike objects of negligible mass is placed on the free end of the rod and another is held against it from below, as shown in the figure. The coefficient of friction between the small objects and the rod is $\mu = 0.841$. The system is released from rest. At what distance from each other will the small objects hit the ground?

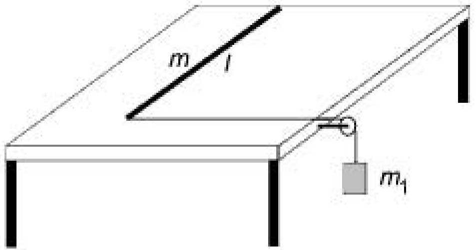
Problem 96. A thin, homogeneous rod of length L and mass m is suspended on a hinge at one end and then displaced into a horizontal position as is shown in the figure. The rod is released without an initial velocity. Find the magnitude and the direction of the force exerted by one half of length $L/2$ of the rod on the other half of $L/2$ when the angular displacement is $\varphi = 60^\circ$.



Problem 97. A disk rotates at constant angular velocity around its vertical axis of symmetry. A rod of length $L = 1\,\mathrm{m}$ is placed onto the disk in a way that its one end touches the disk at a distance of $r = 0.8\,\mathrm{m}$ from the centre, while its other end is above the centre as shown. The rod is then released and rotates together with the disk in this position. Find the angular velocity of the disk. Use $g = 10\,\mathrm{m/s^2}$.



Problem 98. There is a rod of length l, mass m lying on a horizontal table. A cord is led through a pulley, and its horizontal part is attached perpendicularly to one end of the rod, while its vertical part is attached to a weight of mass m_1. The mass of the pulley and the friction are negligible.



a) Which point of the rod has zero acceleration at the moment of releasing the weight?

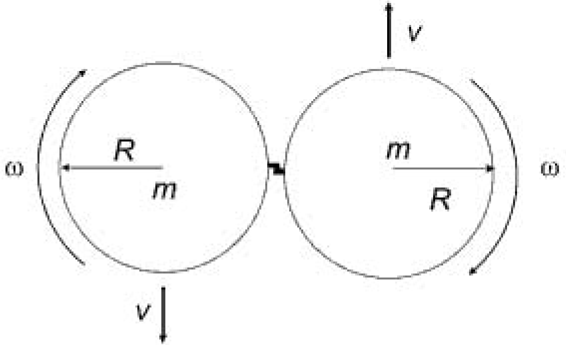
b) At what mass ratio is the acceleration of the centre of the rod maximal at the moment of releasing the weight? Determine this acceleration.

Problem 99. A thin rod of length l, mass m and uniform mass distribution is lying on a smooth tabletop. One end is given a sudden horizontal impulse in a direction perpendicular to the length of the rod. How long will the rod slide along the table as it makes two complete revolutions?

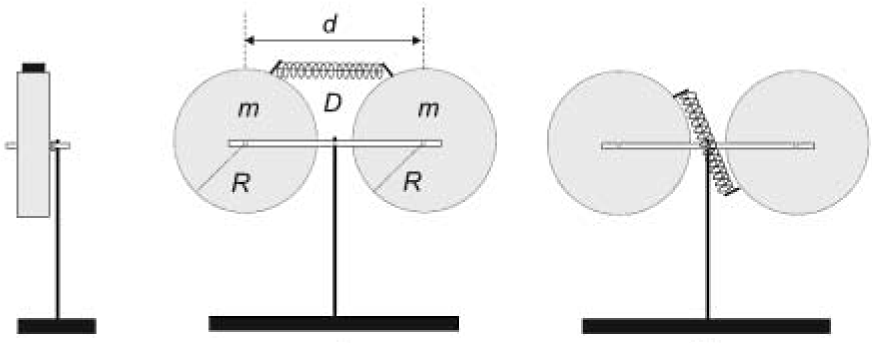
Problem 100. Two discs of radius $R = 4$ cm rotating in the same direction at angular velocity $\omega = 2\ \mathrm{s}^{-1}$ move in opposite directions at velocity $v = 10$ cm/s on an air-cushioned table as shown in the figure. The discs collide along the spikes that are located on their circumferences and whose dimensions are negligible. Determine the velocities after the collision if the discs

a) stick together firmly after a perfectly inelastic collision,

b) part after a perfectly elastic, instantaneous collision.

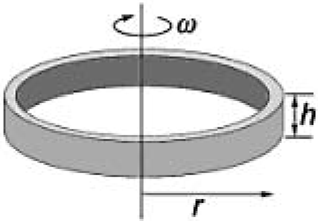


Problem 101. There are two homogeneous, solid disks of radius $R = 10$ cm and mass $m = 4$ kg mounted by two parallel, horizontal axes at the ends of a horizontal rod of negligible mass. The distance between these axes is $d = 25$ cm and the disks can freely rotate around them. The rod itself, with the disks mounted on it, can also freely rotate around a horizontal axis in its midpoint. (See the figures. All the three axes are perpendicular to the rod.) On the rim of each disk there is a small pin, and between them there is a spring of spring constant $D = 1800$ N/m, which is initially compressed by $\Delta l = 5$ cm. Determine the angular velocity of the disks after we burn the thread that holds the spring in its compressed position, provided that its initial position corresponds to figure a) or figure b). (The spring is in contact with the pins until it extends to its unstretched position, and then falls down.)

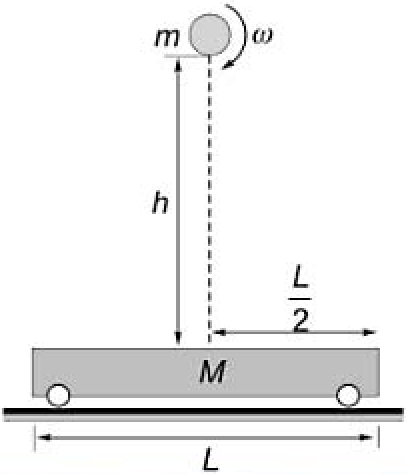


a) b)

Problem 102. A thin ring of radius $r = 10$ cm, rotating in a horizontal plane, is dropped onto a tabletop from a height of $h = 20$ cm. At the instant when it starts to fall, the angular speed of the ring is $\omega_0 = 2\ \mathrm{s}^{-1}$ around its vertical axis. The collision is inelastic and takes a very short time. The coefficient of friction between the ring and the tabletop is $\mu = 0.3$. $g = 10\ \mathrm{m\,s}^{-2}$. How many revolutions will the ring make from the start of its fall until it finally stops?



Problem 103. A solid and rigid sphere of mass $m = 80$ kg and radius $R = 0.2$ m is spun about a horizontal axis at an angular speed of ω, and then dropped without an initial speed onto a stationary cart of mass $M = 200$ kg from a height of $h =$ $= 1.25$ m. It hits the cart exactly at the centre. (The longitudinal axis of the cart lies in the plane of the rotation.) The cart can roll smoothly, its deformation in the collision is perfectly elastic, and the collision is momentary. The sphere keeps sliding throughout the entire duration of the collision. The coefficient of kinetic friction between the sphere and the cart is $\mu = 0.1$. The sphere rebounds from the cart and falls back onto it again.



a) What is the minimum possible length of the cart?

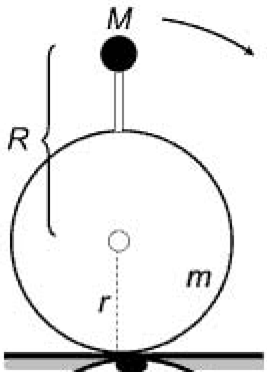
b) What is the minimum possible initial angular speed of the sphere?

c) Provided that the sphere is started at the minimum angular speed as in question b), how much mechanical energy is dissipated in each of the first and second collisions?

d) Find the total work done by the friction force and the works done by the sphere on the cart and by the cart on the sphere.

e) How much translational kinetic energy does each object gain? What is the change of the rotational kinetic energy?

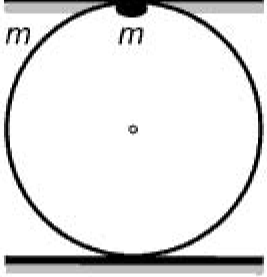
Problem 104. A small ball of mass $M = 4$ kg is attached to a solid cylinder of radius $r = 3$ dm and mass $m = 40$ kg by a massless rod as shown. The ball is at a distance of $R = 5$ dm above the centre of the cylinder. The system is then tipped from its unstable equilibrium position. Find the speed of the ball when it hits the ground. The cylinder rolls without slipping. Use $g =$ $= 10\ \mathrm{m/s^2}$.



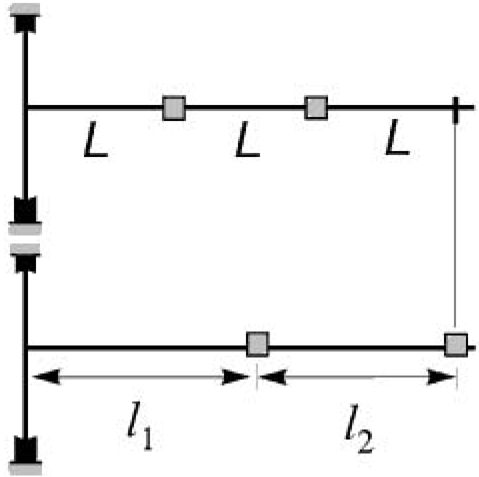
Problem 105. A weight of mass $m = 5$ kg is fixed to the perimeter of a hoop of the same mass $m = 5$ kg and radius $r = 1$ m. The hoop is placed on a horizontal plane. Friction is negligible. $g = 10\ \mathrm{m/s^2}$. Initially, the weight is at the top. Then the hoop is released.

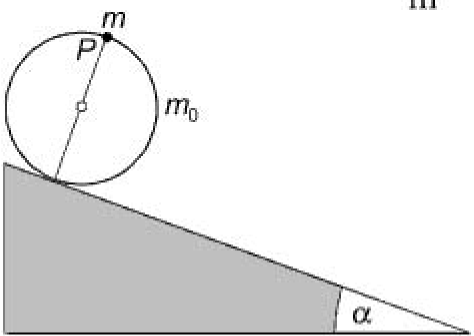
a) Find the acceleration of the centre of the hoop when the weight is level with the centre.

b) With what force does the hoop press on the ground at that time instant?



Problem 106. A horizontal rod is fastened to a vertical axis as shown. There are two identical particles beaded onto the rod, each of mass 1 kg. The particles are connected to each other and to the axis by two springs, each of which have a length of $L = 0.1\ \mathrm{m}$ in their relaxed states. The particles can move on the rod without friction. What should the angular velocity of the system be if the distance of the outer particle from the axis is to be $3L$? The spring constant is $D = 10\ \dfrac{\mathrm{M}}{\mathrm{m}}$.

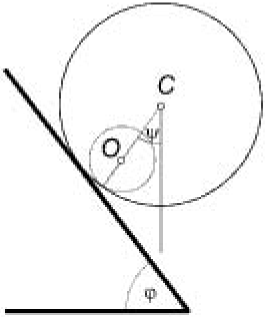




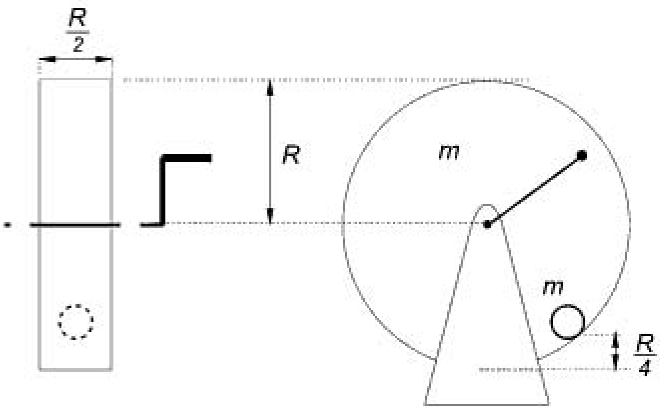
Problem 107. A ring of mass m_0 rolls along a slope with angle of inclination α without sliding. When it begins, a beetle of mass m lands at point P. Find the force with which the beetle should hang on to the ring after $5/4$ turns in order to remain on the ring. ($\alpha = 20°$, $m = 1\,\mathrm{g}$, $m_0 \gg m$.)

Problem 108. A hoop of radius r and of mass m is thrown above the ground in such a way that the plane of the hoop is vertical. The hoop is rotating backwards, about its centre with an angular speed of ω_0 and the velocity of its centre is v_0 in the forward direction. What must the angular speed of the hoop be if after reaching the ground during the course of its motion the hoop turns back (moves backward)? At what angular speed of ω_0 will the speed of the hoop moving backward be v_0?

Problem 109. A ping-pong ball of mass $m = 3$ g is hit back in such a way that it gains a horizontal velocity at a height of $h = 20$ cm above the table. There is a spin put on the ball causing it to rotate about a horizontal axis that is perpendicular to its velocity. After hitting the table the ball bounces back in the vertical direction without rotation. The collision is elastic, and due to the unevenness of the surfaces the coefficient of kinetic friction between the ball and the table is not zero, but $\mu = 0.25$. Therefore, what is the maximum heat produced during the collision of the ball with the table? (Use $g = 10\ \mathrm{m/s^2}$.)



Problem 110. A hollow rim of radius $r_1 = 1$ m and mass $m_1 = 670$ g rolls down an inclined plane of angle $\varphi = 53^0 08'$. Inside the rim there is a solid cylinder of radius $r_2 = 0.3$ m and mass m_2. The centre of mass of the cylinder remains at rest relative to the centre of mass of the rim so that the line connecting the two centres (points O and C) forms an angle $\psi = 36°52'$ with the vertical throughout the motion. Find the mass of the cylinder if both objects roll without slipping.



Problem 111. For a freely rotating wheel of fortune of mass m, the base and the nappe of the cylinder, whose radius is R and height is $R/2$, are made of a plate of uniform width and material. Within the originally stationary wheel there is a solid ball of radius $r = R/6$ and the same mass m, which is in touch with the surface of the cylinder at a height $R/4$ and is initially at rest.

a) Find the torque that should be

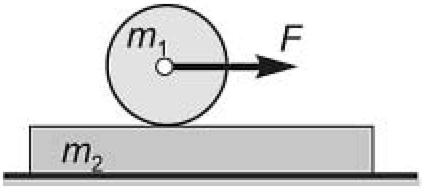
applied on the wheel of fortune in order to have the centre of mass of the ball in it stay at rest.

b) Find the work done this way in 2 s.

c) Find the angular acceleration of the ball and the wheel of fortune.

(The ball rolls without skidding. Let $R = 0.54$ m and $m = 2$ kg. The mass of the driving rod is negligible.)

Problem 112. A cylinder of mass $m_1 = 30$ kg and radius $r = 8$ cm lies on a board of mass $m_2 = 60$ kg. The ground is frictionless and the coefficient of friction (both static and kinetic) between the board and the cylinder is $\mu = 0.1$. The centre of mass of the cylinder is pulled with a force of $F = 44.15$ N for two seconds. Find the work done by force F.

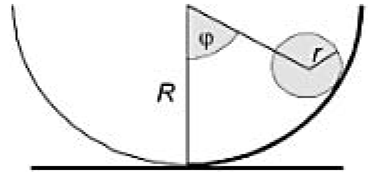


Problem 113. A solid sphere is rolling down, without sliding, on an incline of angle $30°$. The angle of the incline is variable.

a) The experiment is repeated with a hollow sphere, containing a concentric, spherical hole of half radius inside. Determine the slope of the incline so that the time of the motion is the same as in the previous experiment, provided that the two spheres are started from the same point on the incline.

b) In which case is larger the minimal static friction coefficient necessary for the slide free rolling?

Problem 114. One half of a semi-cylinder of radius $R = 1$ m has a rough inner surface, while the other half of the surface is frictionless. A solid sphere of radius $r = 0.2$ m is released from the position described by the initial angle $\varphi = 60°$ on the rough part of the semi-cylinder. Determine how high the centre of the sphere gets on the other, frictionless part of the semi-cylinder, in respect to the lowest point of the circular ramp. (On the rough part of the surface the static friction is strong enough for rolling without slipping, and the rolling friction is negligible.)

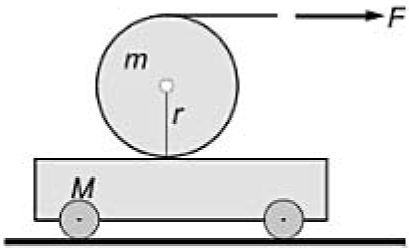


Problem 115. A disk of mass $m = 10$ kg and radius $r = 0.2$ m is placed on top of a cart of mass $M = 5$ kg that stands on a frictionless surface. A massless string is wrapped around the disk.

a) Find the accelerations of the disk and the cart, if the free end of the string is pulled with a constant horizontal force of magnitude $F = 100$ N. The coefficient of friction between the cart and the disk is $\mu = 0.1$.

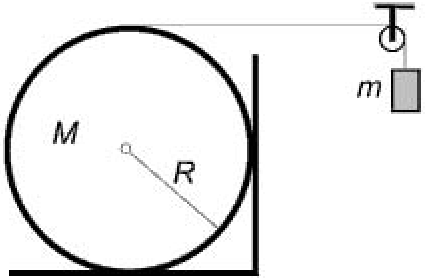
b) Find the kinetic energies of the two objects at the instant when the length of the unwound string is $L = 2$ m.

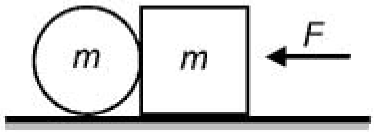
c) Find the work done by force F until that moment.



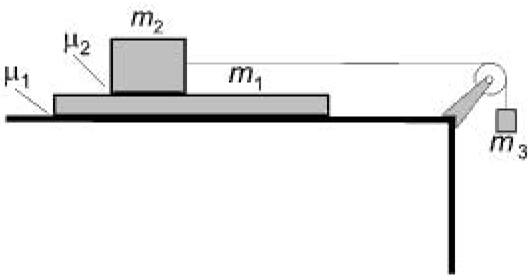
Problem 116. There is a ball of mass m at the middle of the top of a block of mass M and of length $2l$. A constant force of F is exerted on the block from the initial time 0 till time t. Then the exerted force is ceased. Friction between the horizontal surface and the block is negligible. The static friction between the ball and the block ensures that the ball rolls without sliding. Find the time T which elapses until the ball falls off the block. (When will the ball reach the end of the block?) The rolling resistance exerted on the ball is negligible.

Problem 117. A cylinder of mass M and radius R lies in a corner so that it touches both the wall and the ground as shown. A massless chord passes around the cylinder, over a pulley, and is attached to a small object of mass m. The coefficient of kinetic friction is μ for all surfaces. Find the acceleration of the object attached to the string. Data: $\mu = 0.5$, $m = 11\,\mathrm{kg}$, $M = 8\,\mathrm{kg}$, $R = 0.4\,\mathrm{m}$, $g = 10\,\mathrm{m/s^2}$.

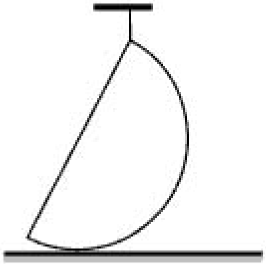




Problem 118. A cube and a cylinder are placed on a horizontal surface such that a generator of the cylinder touches the side of the cube as shown. The radius of the cylinder is equal to the side length of the cube and the masses of the two objects are also equal. For all surfaces the coefficients of static and kinetic friction are μ_0 and μ respectively ($\mu_0 > \mu$). With what force should the cube be pushed if the two objects are to move together in such a way that the cylinder's motion remains purely translational? Data: $m = 12\,\mathrm{kg}$, $\mu = 0.2$, $\mu_0 = 0.6$, $g = 10\,\mathrm{m/s^2}$.

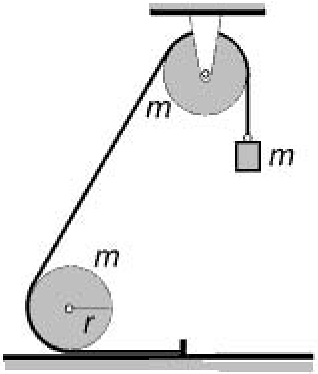


Problem 119. Describe the motion of the system shown in the figure. The coefficient of friction between the board of mass m_1 and the table is μ_1, while the coefficient of friction between the board and the brick of mass m_2 is μ_2. (The coefficients of static and kinetic friction are the same.) Data: $m_1 = 2\,\mathrm{kg}$, $m_2 = 2\,\mathrm{kg}$, $m_3 = 1\,\mathrm{kg}$, $\mu_1 = 0.1$, $\mu_2 = 0.35$.

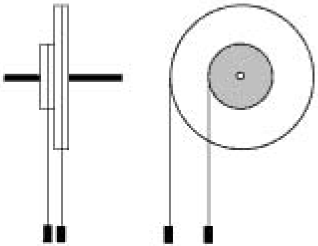


Problem 120. A homogeneous full hemisphere is suspended by a string at a point on its edge is such a way that it touches but does not push the ragged surface beneath it. Find the minimum value of the coefficient of friction at which the hemisphere will not slip after burning the string. The centre of mass of a hemisphere is at $3/8$th of its radius.

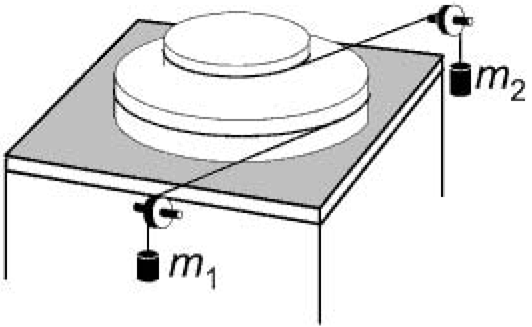
Problem 121. A massless cord that has one of its ends attached to a peg on the ground passes below a cylinder, over a pulley, and is attached to a small object of mass $m = 8\,\mathrm{kg}$ as shown. The rotating part of the pulley is identical to the cylinder on the ground, both having a radius of $r = 25\,\mathrm{cm}$ and a mass of $m = 8\,\mathrm{kg}$. The cord between the pulley and the cylinder (that are at a great distance from each other) forms 60° with the horizontal. Find the acceleration of the hanging object at the moment when it is released. (The cord does not slip on the pulley.)



Problem 122. Two coaxial pulleys of the same thickness and of the same material have radii $10\,\mathrm{cm}$ and $20\,\mathrm{cm}$ respectively. The total mass of the pulley-system is $5\,\mathrm{kg}$. The blocks hanging from the pulleys have a mass of $9\,\mathrm{kg}$ each. Find the times the hanging blocks need to travel down to a depth of $4.9\,\mathrm{m}$ from their original positions.

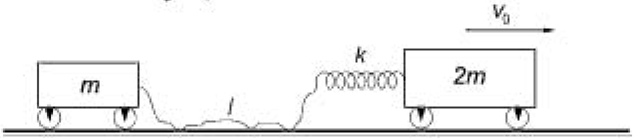


Problem 123. Two disks with radii $r_1 = 0.3\,\mathrm{m}$ and $r_2 = 0.2\,\mathrm{m}$ are fixed together so that their centres are above each other. The rotational inertia of the disk-system is $\Theta = 0.25\ \mathrm{kgm}^2$. The greater disk stands on a frictionless table. Massless chords that are wrapped around the greater and the smaller disks pass over pulleys and are attached to small objects of masses $m_1 = 5\ \mathrm{kg}$ and m_2 respectively as shown. Find the value of m_2 at which the axis of symmetry of the disk-system remains stationary. Use $g = 10\,\mathrm{m/s}^2$.



Problem 124. A cylinder of radius R has two disks, both of radius $r = R/3$ fixed onto its two base surfaces. The system is suspended on two massless chords that are wrapped around the disks. There are inked letters placed all around the cylindrical surface. With what acceleration should the end of the cords be moved if our task is to print the letters clearly onto a vertical wall? Neglect the mass of the disks.

Problem 125. A loosely hanging thread of length l is attached to a freely rolling trolley of mass m; the other end of it is attached to a cylindrical spring with spring constant k whose other end is attached to a trolley of mass $2m$ as shown in the figure. The spring can also be compressed and its axis always remains straight. The cart of mass $2m$ is pushed at velocity v_0.



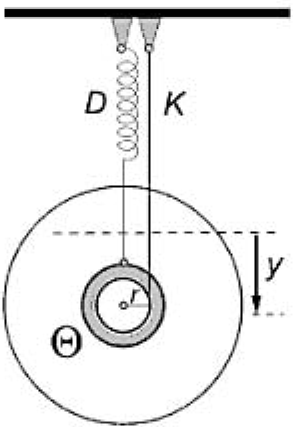
a) Find the time elapsed from the stretching of the thread to the rear trolley reaching the spring.

b) Find the time after which the thread stretches again.

($m = 8\,\mathrm{kg}$, $k = 23.3\,\mathrm{N/m}$, $l = 1\,\mathrm{m}$, $v_0 = 2\,\mathrm{m/s}$.)

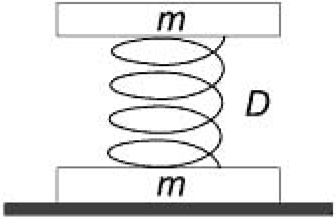
Problem 126. A spring balances a disc of radius R and of moment of inertia Q, which is able to rotate about a horizontal axle, so that the torque exerted by the spring is proportional to the angle turned. A thread which is attached to the spring is wound around the disc and a small body of mass m is hung from its other end. What type of motion will this system undergo if it is moved a little bit out of its equilibrium position? Neglect friction and air resistance.

Problem 127. An axle is attached to a disk at its centre perpendicularly to the plane of the disc. Then two pieces of thread are wound round the two ends of the axle. The ends of the threads are kept vertically and attached to the ceiling, while the disc is held at rest. Symmetrically to the disc two frictionless rings are placed on the axle, and two springs are attached to the rings. The other ends of the springs are fixed to the ceiling so that they hang vertically. The springs are not extended at this position.



Then the system is released. How much time elapses until the disc reaches its lowest position?

(Numerical data: the mass of the disc is $m = 2$ kg, its moment of inertia $\Theta = 0.01\ \mathrm{kgm}^2$, radius of the axle $r = 2\,\mathrm{cm}$, spring constant of one spring (the springs are alike) $D = 1.5\ \mathrm{N/m}$, $g = 10\ \mathrm{m/s}^2$.)



Problem 128. Two slabs of mass $m = 0.1$ kg are connected by a spring of spring constant $k = 20$ N/m, whose unstretched length is $l_0 = 0.3$ m as shown in the figure. The upper slab is pushed down by 0.15 m and then released. Find the maximum distance between the two slabs.

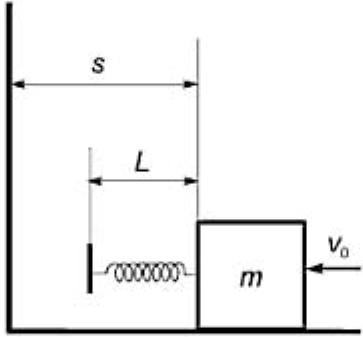
(The mass of the spring is negligible. Calculate with $g = 10$ m/s^2.)

Problem 129. An object hangs from a spring in the cockpit of a truck and causes an elongation of $\Delta l = 0.1$ m of the spring. The truck arrives at a highway that was built from concrete plates of length $x = 20$ m fitted next to each other, but the fittings are not perfect. When the truck runs at speed v, the hanging body oscillates with very high amplitude. What is the speed of the truck?

Problem 130. An object of mass $m = 1\,\mathrm{kg}$ moving on a horizontal ground is given an initial speed of $v_0 = 2\,\mathrm{m/s}$. Initially, the distance of the object from the wall is $s = 1$ metre. A spring of length $L = 8\,\mathrm{cm}$ and spring constant $D = 100\,\mathrm{N/m}$ is attached to the object. The coefficient of friction is $\mu = 0.2$. $g = 10\,\mathrm{m/s^2}$.

a) Where will the block stop?

b) When will the block stop?

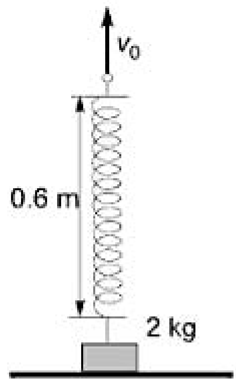


Problem 131. The unstretched length of a spring is $L_0 = 0.6$ metres and the spring constant is $D = 80\,\mathrm{N/m}$. The lower end of the spring is attached to an object of mass $m = 2\,\mathrm{kg}$ lying on the ground, and the upper end is held at a height of 0.6 metres vertically above the object. Initially, the spring is unstretched. Then the upper end is lifted at a uniform speed of $v_0 = 0.5\,\mathrm{m/s}$. $g = 10\,\mathrm{m/s^2}$.

a) How high will the object be lifted in 1.75 seconds?

b) What is the work done by the lifting force?

c) Describe the variation of power as a function of time.



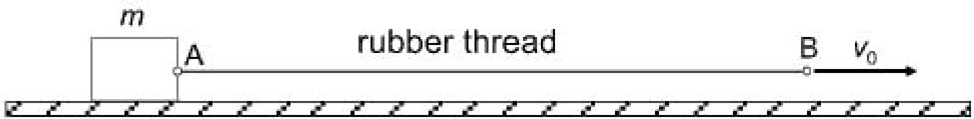
Problem 132. A body of mass $m = 1.25$ kg is suspended vertically by a spring of spring constant $D = 250$ N/m and unstretched length $l = 1$ m. It is released at zero initial speed from the unstretched position of the spring. Determine the time when the speed of the body reaches the value $v = 0.5$ m/s first.

Problem 133. A body of mass $m = 1$ kg is at rest on a horizontal, frictionless ground. A thin rubber thread is attached to the side of the body at the point A. The unstretched length of the thread is $L_0 = 50$ cm. Initially the other end of the thread (point B) is at a distance L_0 from A in horizontal direction. When the rubber thread is stretched, it behaves as if it had a spring constant $D = 100$ N/m, but it is impossible to "compress" the thread, since then it loosens and exerts no force.

At a given moment we start to pull the end B of the rubber thread horizontally, at a constant speed $v_0 = 1$ m/s to the right (see the figure), and continuously maintain this uniform pull.

a) Determine the longest distance between the points A and B.

b) How long does it take for the body to catch up with point B?



Problem 134. A pipe produces a tone of frequency 440 Hz. (This is the frequency of the normal a' above middle c'.) We sound the pipe twice, first normally, by blowing air into it, then by breathing pure helium, and blowing it into the pipe. In both cases the gas flows in the same way in the pipe.

a) Determine the frequency ratio of the two sounds. What is the musical interval?

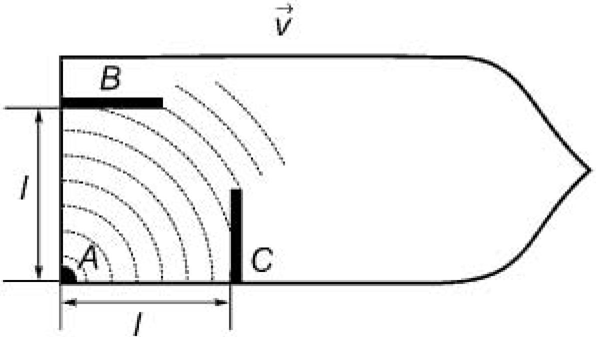
b) How long is this pipe when it is open and when it is closed?

Problem 135. For a wave travelling along a straight line, the difference between two points in the same phase is 5 m, while the distance between two points that are in the opposite phase is 1.5 m. Find the possible values of the wavelength.

Problem 136. During an earthquake the ground is observed to move horizontally. First it moves suddenly 5 cm to the right, then after 1 second it moves suddenly to the left by 5 cm. A chandelier hangs on a $4m$ long cord. Find the amplitude of the chandelier after the earthquake.

Problem 137. A pipe produces sound whose frequency is 440 Hz. (This is the so-called normal sound.) The pipe is sounded twice in such a way that first the gas originating from a container of air, then the gas from a container of helium is 'blown' into it. (The gas flows out of both containers under the same conditions.)
Determine the ratio of the frequencies of the sounds produced by the pipe and the frequency of the sound produced by the pipe "blown" with helium.

Problem 138. Somebody intends to determine the moment of inertia of the first wheel of a bicycle so that a) he totally balances the wheel at its axle (so the wheel stays at rest at any position when its is held at its axle), b) he fixes a point-like lead weight of mass m to the spoke of the wheel at a distance of l from the centre of the rotation, c) he makes the wheel swing, and measures the period of swinging T. Using this data can he find the moment of inertia of the wheel? What is this moment of inertia if $m = 0.5\,\mathrm{kg}$, $l = 0.2\,\mathrm{m}$ and $T = 1.2\,\mathrm{s}$?



Problem 139. A ship swims at constant velocity $\vec{v}$ on a windless ocean. A short sound pulse is emitted from a sound source located at point A of the open deck of the ship. The sound is reflected from wall B that is at distance l from point A and is parallel to the direction of travel. The sound is also reflected from wall C that is also at distance l but is perpendicular to the direction of travel. The reflected sounds arrive back at the sound source with time difference Δt. (This time difference is measured by a timepiece that is connected to a microphone placed next to the sound source.)

Find the speed of the ship if distance l, the speed of propagation of sound (c) and the measured time difference (Δt) are given.

($l = 15\,\mathrm{m}$, $c = 320\,\frac{\mathrm{m}}{\mathrm{s}}$, $\Delta t = 40\,\mu\mathrm{s}$.)

Problem 140. Underneath the topmost, homogeneous covering rock layer which covers the ground and has a horizontal surface, there is another inclined rock plate of different density and composition. The seismic waves generated by an explosion on the surface of the ground are detected at three different places with the help of geophones. The first geophone is at the place of the explosion, and it detected the reflected seismic waves 0.2 s after the explosion. The second geophone is at 50 m east, the third is at 50 m west from the place of the explosion. The second geophone detected the reflected waves with a time delay of 0.26 s, while the third seismic detector measured a delay of 0.34 s.

a) Determine the propagation speed of seismic waves in the topmost, covering rock layer.

b) Determine the distance of the inclined rock plate from the place of the explosion.

c) Determine the angle of inclination of the rock plate in east–west direction.

Problem 141. With what speed can a vehicle move on a planet of uniform density, which is equal to the average density of the Earth, and of radius 500 times greater than that of the Earth. The planet does not rotate. (The radius of the Earth is 6370 km.)

Problem 142. A spaceship moves in a circular orbit of radius r_1 around the Earth with period T_1. Then, with the help of two separate course corrections, the spaceship is put into a new circular orbit of radius $2r_1$. In the first correction only the magnitude of the spaceship's velocity is changed keeping its direction unchanged. In the second correction, which is carried out in the first appropriate moment, only the direction of the spaceship's velocity is changed while its magnitude remains unchanged.

a) By what percentage is the kinetic energy increased during the first course correction?

b) By what angle is the direction of velocity changed during the second correction?

c) Find the time that elapses between the two course corrections. (Assume that the course corrections are carried out instantaneously.)

Problem 143. At what height, measured from the surface of the Earth, does the satellite complete its ninety minute orbit? The Earth is considered to be a sphere with a radius of 3670 km. Assume that the acceleration due to gravity at the surface of the Earth is known ($g = 9.81\,\mathrm{m/s^2}$). The orbit of the satellite is circular.

Problem 144. A spy satellite, travelling above the equator of the Earth, is taking pictures. Assuming that in six hours the satellite is ready with pictures around the whole equator, determine the altitude of the orbit.

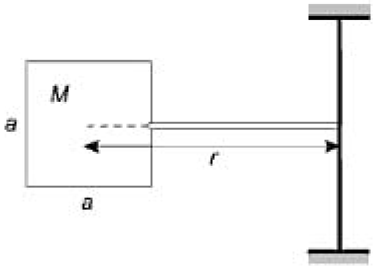
Problem 145. An astronaut revolves around the Earth along a circular path while facing the same point of the Earth all the time. For which points on the Earth can this condition hold true? What is the speed of the revolving spaceship?

Problem 146. How can the mass of an object be determined in a spaceship orbiting the Earth? The engines of the spaceship are shut off and air resistance is negligible. Find as many different ways as you can and describe the methods and equipment used. Which of these equipments need to be calibrated in advance?

Problem 147. A satellite follows a circular orbit around a planet, whose period of revolution is $T_1 = 8$ h. As it wishes to change over to another circular orbit whose period of revolution is $T_2 = 27$ h, it makes a course correction. First it changes the magnitude of its velocity by switching on the rockets for a short period of time and orbiting on a transitional elliptical orbit. When it reaches the desired altitude, it switches on the propulsion again and changes over the circular orbit with period of revolution T_2 solely by changing the magnitude of its velocity.

a) Find the time required for the course correction.

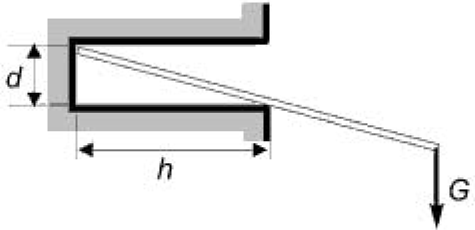
b) Find the percentage change in the magnitude of the velocity of the satellite caused by the switching on of the rockets in the first and second steps within the context of the non-rotating reference frame fixed to the planet.



Problem 148. A 100%-reflexive square mirror is attached to a horizontal massless rod that is attached to an axis of rotation supported in the vertical position by bearings as shown in the figure. The mass of the mirror is $M = 20\,\mathrm{g}$, the side of the square is $a = 10\,\mathrm{cm}$. The centre of the square is $r = 20\,\mathrm{cm}$ from the axis of rotation. Intense sunlight shines on the mirror at right angles, which delivers $0.125\,\mathrm{J}$ energy on each cm^2 of the surface of the mirror in $1\,\mathrm{s}$.

The apparatus starts rotating due to the light pressure. Find the angular displacement which takes place in 1 minute, if the system can move freely, and if it is ensured that light propagates at a right angle to the mirror in each phase of the rotation. (The relationship between the energy and the momentum of the photon is $E = p \cdot c$, where c is the speed of light.)

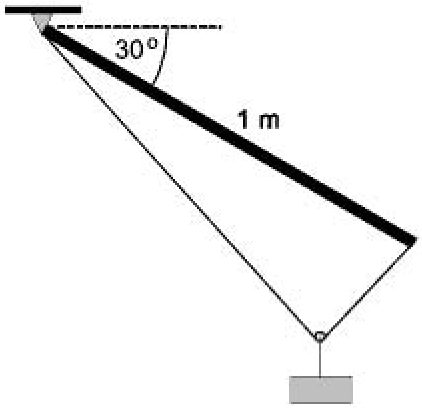
Problem 149. A double-armed lever has equal arms on both sides. One end of the lever has a pulley of negligible mass fixed on it by a hinge, while the other end has a block of mass m_0 suspended from it. Two blocks of masses m_0 and m are attached to each end of a string that runs around the pulley. Find the value of m at which the lever remains in its horizontal position.



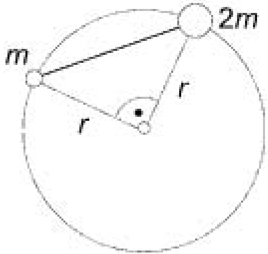
Problem 150. A $h = 6$ cm deep hole of diameter $d = 2$ cm is drilled into a wall. A thin rod of negligible mass is then placed into the hole as shown. The coefficient of friction is $\mu = 0.2$. What is the shortest possible length of the rod if it is to be used as a coat-hanger?

Problem 151. One end of a rod of mass 1 kg and of length 1 m can freely rotate about a fixed horizontal axis. Initially the rod makes an angle of 30° to the horizontal. A thread of length 1.3 m is attached to the two ends of the rod. A small pulley can run without friction along the thread, and a weight of mass 0.2 kg is suspended on the axis of the pulley.

Determine the work needed to lift the rod to a horizontal position. (Use the value $g = 10\ \mathrm{m/s^2}$ for the acceleration due to gravity.)

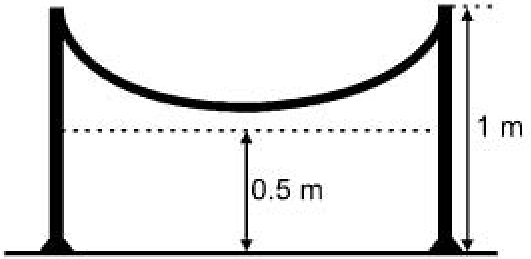


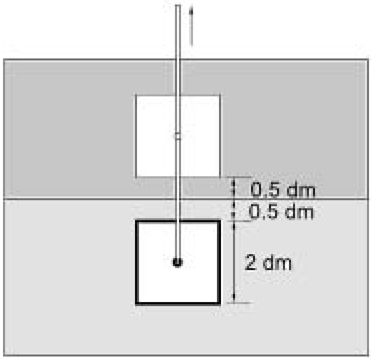
Problem 152. Two beads of masses m and $2m$ can move on a circular vertical loop of radius $r = 0.5$ m. The beads are connected by a massless string, and if the string is taut, it keeps the beads on the ends of a quarter-circle as shown. The coefficient of friction is 0.15. Find the positions in which the beads are in equilibrium with the string being taut.



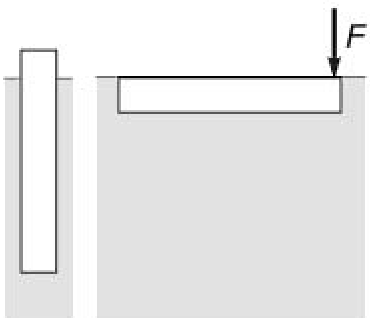
Problem 153. An analytical balance is used with brass weights. Find the mass of a body made of Plexiglas, whose two measurements result in a difference of at least one mark if one measurement is performed in dry weather and the other in wet weather? In both cases the room temperature is 23 °C and the atmospheric pressure is 10^5 Pa. In wet weather the pressure of the water vapour in the air is $2 \cdot 10^3$ Pa. The sensitivity of the balance is 0.1 mg/scalemark. ($\varrho_{Cu} = 8.5 \cdot 10^3\ \mathrm{kg/m^3}$; $\varrho_{Plexiglas} = 1.18 \cdot 10^3\ \mathrm{kg/m^3}$.)

Problem 154. The two ends of a homogeneous chain of mass 2 kg are fixed to columns of height 1 m as shown in the figure. The chain is clutched in the middle and is pulled down until it becomes tight. In the meantime 0.5 J of work is done. The lowest point of the chain is then 0.5 m from the ground then. Where was the centre of mass of the chain initially?





Problem 155. A thick layer of oil with density $0.8\,\mathrm{g/cm^3}$ is placed at the top of the water in a tank. The area of the base of the tank is very big. A cube made of magnesium is placed in the tank in such a way that its top face is $0.5\,\mathrm{dm}$ below the boundary. The edge of the cube is $2\,\mathrm{dm}$ and its density is $1.7\,\mathrm{g/cm^3}$. The cube is then pulled up so that its bottom face is $0.5\,\mathrm{dm}$ above the boundary. How much work was performed?



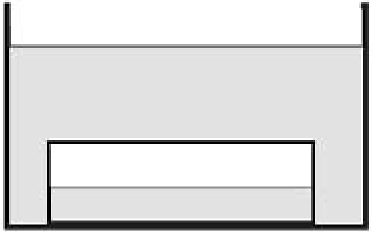
Problem 156. A cuboid shaped piece of wood of base area $1\,\mathrm{dm}^2$ and height $4\,\mathrm{m}$ is floating vertically in a pond, because its centre of mass is not in its geometric centre. To make the wood submerge and float in a horizontal position as shown, we need to exert a downward force of $F = 80\,\mathrm{N}$ at its end. Where is the centre of mass of the wood? Find the work done to the wood by moving it from its first to its second position.

Problem 157. A container is filled with water, and a plank of width 10 cm and of density $\varrho_0 = 0.5\ \mathrm{g/cm^3}$ floats on the surface of the water. Through the tube air at a pressure of 100 atmosphere is compressed into the container. What is the height of that part of the plank which is submerged into the water. (Assume that water is incompressible.) The density of air at a pressure of 1 atmosphere is $0.0013\,\mathrm{g/cm^3}$.

1.4 Fluids

Problem 158. a) A solid sphere of radius $R = 0.2$ m and of negligible mass is swimming on the surface of a lake of depth $h = 1$ m. The sphere is slowly pushed under the water, down to the bottom of the lake. Determine the work done by the external pushing force in the process.

b) Now the sphere investigated in question a) is swimming in a water tank of base surface area $A = 0.5\ \mathrm{m^2}$. The depth of the water in the tank is $h = 1$ m. Determine the work needed to push the sphere down to the bottom of the tank. (Assume that no water flows out of the tank. The density of water is $\varrho = 1000\ \mathrm{kg/m^3}$, and $g = 9.81\ \mathrm{m/s^2}$.)

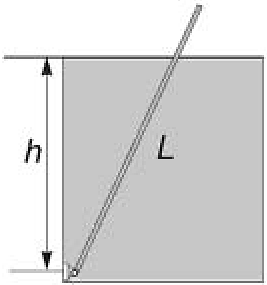


Problem 159. A square based cuboid shaped metal container, whose mass is $13\,\mathrm{kg}$, has a height of $6\,\mathrm{dm}$, a base edge of $2\,\mathrm{dm}$, and is half full of water. The container is laid on its side at the bottom of a cuboid shaped tank, whose base area is $20\,\mathrm{dm}^2$ and in which the level of water is at a height of $4\,\mathrm{dm}$. Find the total work that is required to stand the metal container upright on its square base.

Problem 160. One end of a thin rod of length $L = 1$ m and density ϱ connects to a hinge at a depth of $h = 0.8$ m below water level. Find the equilibrium positions of the rod and state whether the equilibrium is stable or unstable if

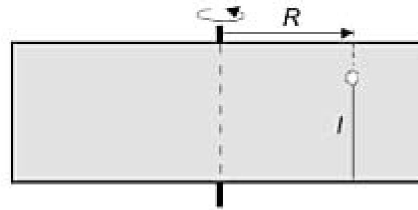
a) $\varrho = 500\,\mathrm{kg/m^3}$,

b) $\varrho = 853\,\mathrm{kg/m^3}$.



Problem 161. A cuboid shaped container has two wheels attached to its bottom and a massless string connected to its side so that it passes over a pulley and attaches to a small object of mass 1.2 kg as shown. The length, height and width of the container are 20 cm, 10 cm and 10 cm respectively, and the height of the water in the container is 9 cm. The mass of the cart and water is 2 kg. Describe the motion of the system. Neglect friction.

Problem 162. A closed cylindrical container with a vertical axis is completely filled with water. A plastic bead of density $\varrho = 0.5\ \mathrm{kg/dm^3}$ and radius $r = 1$ cm is placed at distance $R = 20$ cm from the axis and is anchored to the bottom of the container by a thin thread of length $l = 16$ cm. If as a result of the containers revolutions, the beads sink by $h = 4$ cm, how many revolutions around its axis of symmetry does the container have to make? (In the final state the total content of the container rotates at the same angular speed. Calculate with $g = 10\ \mathrm{m/s^2}$.)



Problem 163. A cylindrical container whose base area is $A = 10\ \mathrm{cm^2}$ contains a $h = 60$ cm high water column.

a) Find the increase in the hydrostatic pressure at a height of $h_1 = 20$ cm above the bottom of the container if the temperature of the water column is increased by $\Delta t = 80\ ^\circ\mathrm{C}$.

b) Give the value of pressure increase as function of distance x measured from the bottom of the container.

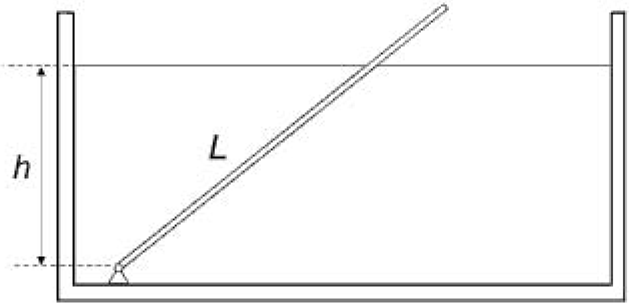
(For water the mean coefficient of expansion is $\beta = 0.00013\ 1/^\circ\mathrm{C}$, the density of cold water is $\varrho = 10^3\ \frac{\mathrm{kg}}{\mathrm{m^3}}$, the expansion of the container is negligible.)

Thermodynamics Problems

2.1 Thermal expansion

Problem 164. The upthrust exerted on a steel ball which is immersed in paraffin of temperature $20\ ^\circ\mathrm{C}$ is $0.2145\,\mathrm{N}$, and $0.200\,\mathrm{N}$ when the temperature of the paraffin is $100\ ^\circ\mathrm{C}$. Based on this measurement, find the volumetric thermal expansion coefficient of paraffin if the coefficient of linear thermal expansion of steel is $1.2\cdot 10^{-5}\ 1/^\circ\mathrm{C}$.

Problem 165. A solid brass sphere rotates freely around an axis which goes through its centre. By how much may its temperature change provided that its frequency does not change by more than 1%? (All frictional effects are negligible.)



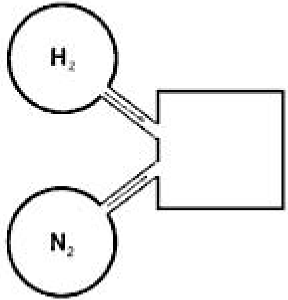
Problem 166. A rectangular glass tank of large base area contains water to a height of $h_0 = 0.6\,\mathrm{m}$. The closed lower end of an aluminium tube of length $L_0 = 1\,\mathrm{m}$, exterior cross-sectional area $A_e = 1.2\,\mathrm{cm}^2$ and interior cross-sectional area $A_i = 1\,\mathrm{cm}^2$ is fixed to the bottom of the tank by a hinge. The initial temperature of the whole system is $t = 4\ ^\circ\mathrm{C}$.

How much will the angle enclosed by the tube and the horizontal change if the temperature of the whole system is raised to 94 $^\circ\mathrm{C}$? (Further data: the coefficient of linear expansion of aluminium and its density are $\alpha_{\mathrm{Al}} = 2.4 \cdot 10^{-5}\ ^\circ\mathrm{C}^{-1}$ and $\varrho_{0_{\mathrm{Al}}} = 2.7 \cdot 10^3\,\mathrm{kg/m}^3$, the coefficient of linear expansion of glass is $\alpha_{\mathrm{glass}} = 8 \cdot 10^{-6}\ ^\circ\mathrm{C}^{-1}$. The mean coefficient of volume expansion of water in this temperature interval is $\beta_{\mathrm{w}} = 4.4 \cdot 10^{-4}\ ^\circ\mathrm{C}^{-1}$. The buoyancy of air is negligible.)

Problem 167. Air at a pressure of 1 atmosphere is confined within a syringe of volume 20 cm^3. Formerly a sample of porous material was placed into the syringe. Find the volume of the porous material if the pressure inside the syringe increases to 2.2 atmospheres when the piston of the syringe is pushed till the mark of 10 cm^3.

2.2 Ideal gas processes

Problem 168. For an experiment a mixture of gases containing 50 volume percents hydrogen and 50 volume percents nitrogen should be continuously provided at a speed of 0.5 kg/min. The cross section of the gas tubes is 10 cm^2. Determine the speed of gas flow in the tubes, provided that the pressure is 10^5 Pa and the temperature is 27 °C in the tubes.



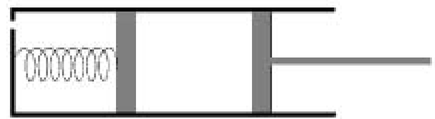
Problem 169. A cylinder with base area 1 dm^2 lies on its side on a horizontal surface and is divided into two parts of volumes 0.8 litre and 4.2 litre by a frictionless vertical piston as shown. The pressure in each part is 0.02 N/cm^2. The masses of the cylinder and piston are 0.8 kg and 0.2 kg respectively. The cylinder is then pushed by a constant horizontal force of magnitude 2.5 N to the left. What will the new position of the piston be? (Assume constant temperature.)

Problem 170. A cylinder of base area $10\,\text{cm}^2$ in which a $47\,\text{cm}$ high air column is enclosed by a piston is floating upside down in a container. The piston is connected by a cord to the bottom of the container, which is filled with mercury and has a base area of $20\,\text{cm}^2$. The closed end of the cylinder is $10\,\text{cm}$ below mercury level.

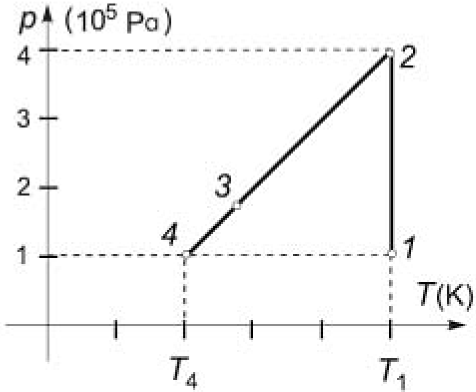
a) Find the new position of the cylinder if the cord is shortened by $6\,\text{cm}$.

b) Find the volume of mercury that should be poured into the container to set the mercury level back into its original height.

Problem 171. The cylindrical vessel shown in the figure has two pistons in it. The piston on the left touches a spring attached to the wall of the vessel. The wall has a hole in it. The volume of the air between the pistons is $2000\,\text{cm}^3$ and its pressure is initially equal to the external atmospheric pressure of $10^5\,\text{N/m}^2$. The piston on the right is slowly pressed inwards, maintaining constant temperature, until its inner surface is at the position where the inner surface of the piston on the left was initially. What will be the final volume of the air between the pistons?

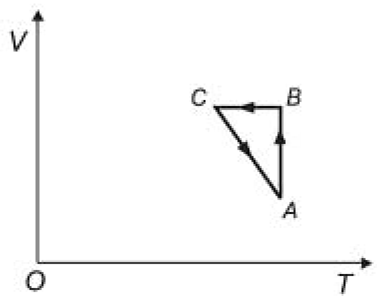


The cross-sectional area of the cylinder is $100\,\text{cm}^2$, and a force of $10\,\text{N}$ compresses the spring by $1\,\text{cm}$.

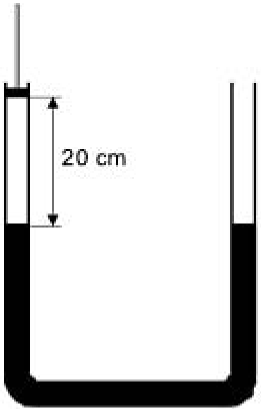


Problem 172. An ideal gas undergoes the process shown in the figure. $T_1 = 500\,\mathrm{K}$, $T_4 = 200\,\mathrm{K}$, $p_1 = 10^5\,\mathrm{Pa}$ and $p_2 = 4 \cdot 10^5\,\mathrm{Pa}$. In state 3, $3V_3 = V_1$. What is the pressure of the ideal gas in state 3?

Problem 173. Using ideal gas we perform the thermal cyclic process $ABCA$, shown in the figure. Found on the volume-temperature plane of the graph $(V;T)$ is a right triangle with legs parallel to the V and T axes. In the state A the temperature of the gas is 373 K and its volume is $5\,\mathrm{dm}^3$, while in the state C the gas has a temperature 273 K and a volume $12\,\mathrm{dm}^3$.



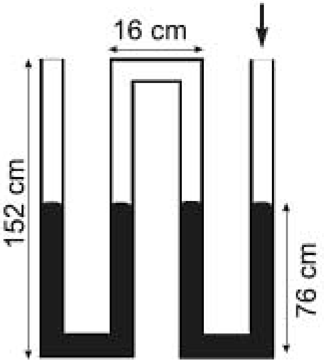
At which volume on the subprocess $C \rightarrow A$ does the gas have the same pressure as in the state B?



Problem 174. One arm of a communicating vessel containing mercury is closed by a piston 20 cm above the mercury. The other arm is open. The mercury level is the same in both arms, whose cross-sectional areas are 2 cm^2. In an isothermal process the piston is pushed down by 10 cm.

a) Find the difference of the mercury levels in the new position of the piston.

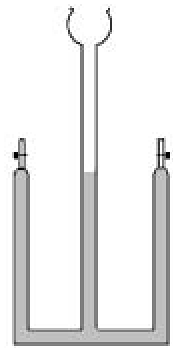
b) Find the change in the energy of the mercury.



Problem 175. Initially, the height of the mercury column is the same in each branch of the narrow glass tube. Atmospheric pressure balances 76 cm of mercury. Then air pressure is increased over the right end until it equals the pressure of 232.8 cm-height of mercury. What is the height of the mercury now in each branch?

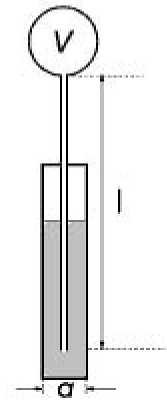
Problem 176. A glass tube is closed at one end and has a cross-sectional area of $0.2\ \mathrm{cm}^2$. The tube is held in a vertical position with its open end facing upwards. It contains a 0.25-cm column of liquid ether that is closed off by a 19-cm column of mercury. The temperature is 35 °C (the boiling point of ether). What will be the position of the mercury column if the tube is inverted? The density of liquid ether is $0.7\ \mathrm{g/cm^3}$ and its relative molar mass is 74.

Problem 177. The cross-sectional areas of all three branches of a device for the electrolysis of water are $A = 4\ \mathrm{cm}^2$. Initially, the height of the water column is the same in every branch, and there is no air above the water in the branches on the sides. How long does it take for the water level to rise by $\Delta h = 1\ \mathrm{m}$ in the middle tube if the device extracts 0.6 mg of hydrogen per minute? What is the (average) speed of the rising water level? The temperature is 27 °C. ($g = 9.8\ \mathrm{m/s}^2$, external air pressure is $p_0 = 10^5\ \mathrm{Pa}$, $\varrho = 10^3\ \mathrm{kg/m}^3$.)



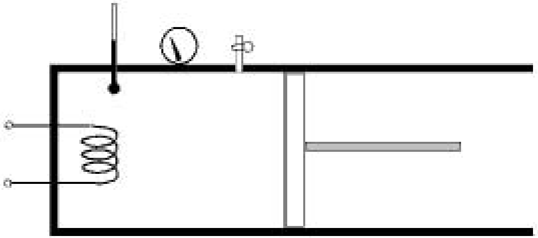
Problem 178. A glass balloon of volume $V = 1\ \mathrm{dm}^3$ is attached to a thin-walled tube of length $l = 40$ cm and cross-sectional area $A = 1\ \mathrm{cm}^2$. The glass tube is immersed into mercury to half of its length as shown in the figure. The container containing the mercury is a square prism with base edges of $a = 3$ cm. At the initial temperature $t_1 = 10\ °\mathrm{C}$ the level of mercury in the tube is the same as the level of the external mercury.

Find the temperature at which the enclosed air should be heated in order to push out the mercury from the tube? (The external atmospheric pressure is $p_0 = 10^5$ Pa, the thermal expansion of the mercury and the glass can be neglected.)



Problem 179. In a container with heat insulator walls a heat insulator piston encloses a diatomic gas at pressure $p_1 = 139.2$ kPa. We turn on an electric heater inside the container, and slowly let the piston extend in such a way that the pressure inside the container remains constant. After some time the temperature of the gas is increased by $\Delta T_1 = 29.3$ °C, while its volume is increased by $\Delta V_1 = 5$ dm^3. Then we turn off the heater and let $\Delta m = 5$ g gas stream out of the container. Thus, the pressure decreases to $p_2 = 130.5$ kPa, but the temperature of the gas remains unchanged.

Now we turn on the heater, and again maintaining constant pressure by the piston we let the volume grow by $\Delta V_2 = 8$ dm^3, while the temperature increases by $\Delta T_2 = 46.88$ °C.



a) Determine the initial mass of the gas in the container.

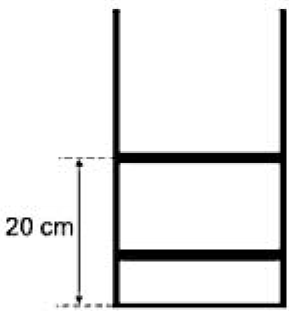
b) What kind of gas is in the container?

c) How much energy did the heater transfer to the gas during the first extension process?

Problem 180. A long glass pipe of cross section $A = 3\ \mathrm{cm}^2$, which is closed at one end, is partially submerged into the water of a lake in such a way that the open end of the pipe points vertically downwards. When the length of the air column in the pipe is $l_0 = 60\ \mathrm{cm}$, the levels of the water in the pipe and in the lake coincide. Then the pipe is slowly pulled out of the water until the water level inside it rises by $h = 50\ \mathrm{cm}$. At that time the length of the air column is $l_1 = 63\ \mathrm{cm}$.

a) Determine the external air pressure.

b) Now the pipe is held fixed at its last position, and the initial temperature of the air in it is $5\,^\circ\mathrm{C}$. By how many degrees $^\circ\mathrm{C}$ should this temperature be increased in order for the water level in the pipe to decrease by $16\ \mathrm{cm}$?



Problem 181. In a cylindrical container of height $40\ \mathrm{cm}$ two pistons enclose certain amounts of gas, as is shown in the figure. The upper piston is at a height of $20\ \mathrm{cm}$ from the bottom of the cylinder. If the upper piston is slowly lifted by $10\ \mathrm{cm}$, then the lower piston rises by $4\ \mathrm{cm}$. Determine the position of the lower piston if the upper one is removed from the cylinder. The external air pressure is $10^5\ \mathrm{Pa}$, the cross section of the cylinder is $10\ \mathrm{cm}^2$ and each piston has a mass of $1\ \mathrm{kg}$. (Assume that the temperature is constant during the process.)

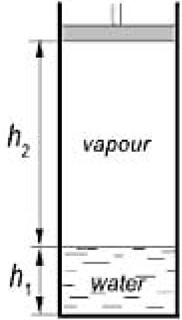
Problem 182. Determine the specific heat of air at constant volume, given the information that 75.5% of air is nitrogen, 23.2% is oxygen and 1.3% is argon, and the atomic masses of nitrogen, oxygen and argon are 14 u, 16 u and 40 u respectively.

Problem 183. There was a block of ice in an isolated container at temperature 0 °C. We wanted to determine its mass, therefore we let some steam with a temperature of 100 °C into the container, but we could not determine the exact amount of the steam. After re-closing the container all the ice melted, and the new equilibrium temperature became 10 °C. Then, once more we let some steam of temperature 100 °C into the container, but again, we could not determine its exact amount. The temperature in the container became 15 °C. Finally, we let again some steam into the container, and this time we could determine its mass, which was 0.3 kg. The new equilibrium temperature in the container became 23 °C. Determine the mass of the ice block.

Problem 184. In an isolated container, which has cooling tubes built in the walls, an amount of $m = 18$ kg of clean water is very carefully cooled down to the temperature of $t_1 = -9$ °C. After this a small ice crystal of negligible mass is thrown into the water, which starts to freeze the super cooled water. Determine the amount of ice produced.

(The necessary material constants should be looked up in a table.)

Problem 185. A vertical cylinder with cross-sectional area $A = 1\ \mathrm{dm}^2$ contains $h_1 = 25$ cm of water at the bottom. The space above it is filled with the saturated vapour of the water, which is separated from the external space by a piston. The bottom of the piston is $h_2 = 75$ cm above the water level. The density of water at this temperature is $n = 2$ times the density of saturated vapour.

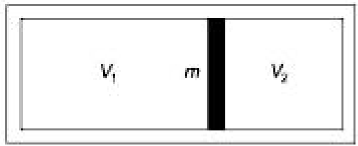


a) If temperature is held constant, by how much should the piston be pushed down in order to decrease the volume of vapour to $V = 4.5\ \mathrm{dm}^3$?

b) If temperature is held constant, by how much should the piston be pushed down in order to have the vapour condense completely?

(The sum of the masses of water and vapour is constant throughout the process.)

Problem 186. Helium gas whose volume is $V_1 = 3$ litres, pressure is $p_1 = 4 \cdot 10^5$ Pa and temperature is $T_1 = 1092$ K is separated from helium gas whose volume is $V_2 = 2$ litres, pressure is $p_2 = 2.5 \cdot 10^5$ Pa and temperature is $T_2 = 1365$ K by a highly insulated wall of mass $m = 2$ kg in an insulated cylinder. The partition wall is released, it can move without friction. Find the maximum speed acquired by the partition wall.



Problem 187. 4 grams of helium and 16 grams of oxygen are enclosed by a piston in a cylinder. The a temperature of the gas is 0 °C and its pressure is 10^5 Pa. The cylinder walls and the piston are good thermal insulators. The pressure is increased to $2\cdot10^5$ Pa. What will be the final temperature and volume of the gas? The molar specific heats of helium are $C_{vh} = 12.3$ J/(mod·K), $C_{ph} = 20.5$ J/(mol·K); and those of oxygen are $C_{vo} = 20.5$ J/(mol·K), $C_{po} = 28.7$ J/(mod·K).

Problem 188. A smoothly moving, fixed piston made of good insulating material separates two gases in an insulated cylinder whose cross-sectional area is $A = 1\ \mathrm{dm}^2$. One part contains helium, the other part contains hydrogen. The initial data of helium is: its pressure is $p_1 = 2\cdot10^5$ Pa, its volume is $V_1 = 4\ \mathrm{dm}^3$, its temperature is $T_1 = 350$ K, the corresponding data of hydrogen are: $p_2 = 3\cdot10^5$ Pa, $V_2 = 5\ \mathrm{dm}^3$ and $T_2 = 280$ K. Find the displacement of the piston when it is released and it reaches an equilibrium

a) if the piston does not allow the gases to mix,

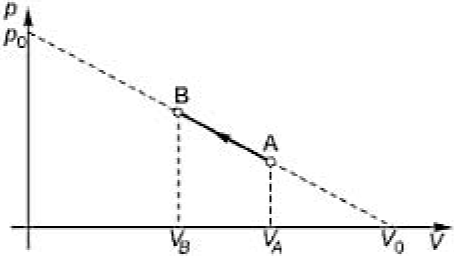
b) if the piston is permeable and particles can diffuse through it slowly,

c) if the piston allows only the helium to diffuse through it.

Problem 189. Three identical containers, each containing 32 g of oxygen gas at a temperature of 200 °C and pressure of $10\ \mathrm{N/cm}^2$ are connected by thin tubes. The container on the left is then cooled down to 100 °C, the one on the right is heated to 300 °C, while the temperature of the middle one remains 200 °C.

a) Find the new pressure of the system.

b) Find the change in the total internal energy of the oxygen gas. The specific heat of oxygen is $c_v = 670\ \mathrm{J/(kg \cdot K)}$.



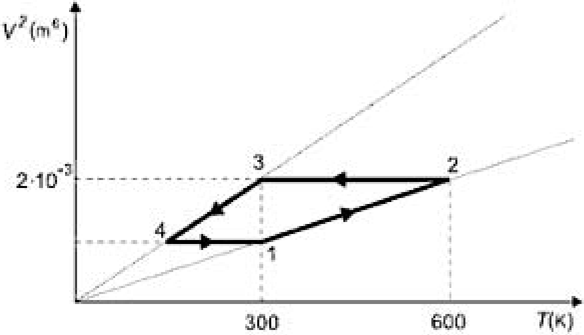
Problem 190. The figure shows the $p(V)$ diagram of a process carried out with a certain quantity of oxygen gas. The values of the volume V_0 and pressure p_0 in the figure are $V_0 = 12\,\mathrm{dm}^3$, $p_0 = 1.2 \cdot 10^5\,\mathrm{Pa}$. In the initial state (A), the volume of the gas is $V_A = \dfrac{2}{3}V_0$ and its temperature is $T_A = 300\,\mathrm{K}$. In the final state (B), $V_B = \dfrac{5}{12}V_0$. Determine the heat absorbed and, separately, the heat given off by the gas during the process.

Problem 191. 2 g of hydrogen gas of volume 22.4 litre at 0 °C and 10^5 Pa is taken through a cyclic process. First, it is slowly heated at constant volume until its pressure reaches $2 \cdot 10^5$ Pa. Then, it is heated to 546 °C at constant pressure. Finally, it is taken back to its original state along a path whose graph is a straight line segment on the p-V diagram. The specific heat of hydrogen is $10.1\ \mathrm{kJ/(kg \cdot K)}$ at constant volume, while at constant pressure it is $14.28\ \mathrm{kJ/(kg \cdot K)}$.

a) Find the efficiency of this cycle.

b) How does the temperature change in terms of the volume and in terms of the pressure through one complete cycle?

Problem 192. The cyclic process shown in the figure is carried out with 1 mole of diatomic gas. Find the percentage of the heat absorbed by the gas that is converted into useful work.



Problem 193. A closed container contains diatomic gas at temperature T_1 and pressure p_1. The gas is then heated to temperature T_2, during which 20% of the molecules are broken into atoms.

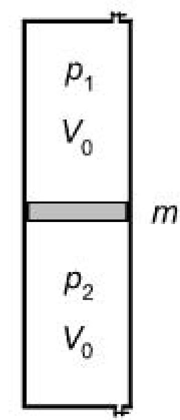
a) Find the final pressure of the gas.

b) Find the ratio of the final and initial internal energies of the gas. (Neglect the oscillation of the molecules.)

Problem 194. 1 mol of He is enclosed by a piston in a heatable and coolable container at an initial volume of $30\,\mathrm{dm}^3$ and an initial temperature of $5\ ^\circ\mathrm{C}$. From this initial state the gas is compressed in such a way that ratio $\Delta p/\Delta V$ remains constant during the process. Find this ratio if during the compression process the maximum temperature of the gas is $71\ ^\circ\mathrm{C}$.

Problem 195. Let us model the atmosphere of the Earth, which has a radius $R_E =$ $= 6370$ km, in the following way: the temperature of the atmosphere is the same everywhere. The air molecules have 5 thermodynamical degrees of freedom, and their average molar mass is $M = 29$ g/mol. The air pressure at the surface of the Earth is $p_0 = 100$ kPa. Furthermore, the acceleration due to gravity is $g = 10$ m/s^2 in the region of the atmosphere. Now let us assume that for some reason the temperature of the atmosphere increases everywhere, uniformly by $\Delta T = 1°$C, but the total mass and the composition of the atmosphere remains unaltered. Determine the increase of the (gravitational) potential energy of the atmosphere due to the temperature change.

Problem 196. A piston of cross section $A = 100$ cm^2 and of mass $m = 0.5$ kg moves freely (without friction) in vertical direction in an isolated cylinder with negligible heat capacity. The specific heat of the material of the piston is $c = 210$ J/(kg °C). Initially the temperature of the piston is $t_0 = 100°$C, and there is an amount of $n_1 = 0.05$ mol noble gas at temperature $t_1 = -90$ °C above the piston, and an amount of $n_2 = 0.03$ mol air at temperature $t_2 = 46$ °C under the piston. The initial volumes of the gases are just equal.



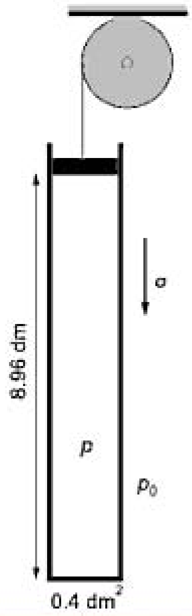
a) Determine the final temperature of the piston.

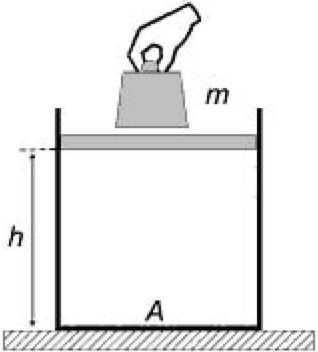
b) Determine the displacement of the piston.

Problem 197. The closed cabin of a space station orbiting around the Earth is filled with artificial atmosphere that contains oxygen gas at pressure $p = 50$ kPa and temperature $T = 295$ K. The internal volume of the cabin is $V = 80\ \mathrm{m}^3$. A tiny hole of area $A = 0.1\ \mathrm{mm}^2$ appears on the wall of the cabin and the oxygen starts to escape. Estimate the time required for the pressure to decrease by 1 % in the cabin.

(The heating system maintains a constant temperature inside the cabin.)

Problem 198. A cylinder of mass 25 kg contains helium gas, which is enclosed by a well-fitted piston of mass 25 kg. The cross-sectional area of the cylinder is $0.4\ \mathrm{dm}^2$ and the piston is at a height of 8.96 dm from the base of the cylinder. The piston is attached to a massless string that is wrapped around a pulley of radius 0.2 m and rotational inertia $3 \cdot \mathrm{kg} \cdot \mathrm{m}^2$. The container and piston move downwards with the same constant acceleration. The atmospheric pressure is $p_0 = 10\ \mathrm{N/cm}^2$, the temperature is 0^0 C and $g = 10\ \mathrm{m/s}^2$. Find the mass of the helium gas. (Neglect friction)



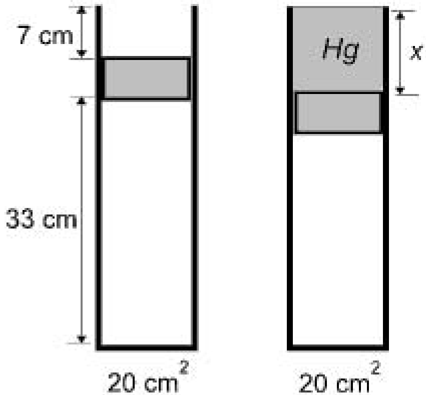


Problem 199. A certain amount of air is enclosed into a vertical cylinder of base surface $A = 1\ \mathrm{dm}^2$ by a frictionless piston of negligible mass. The height of the air column in the cylinder is $h = 5\ \mathrm{dm}$. We carefully put a weight of mass $m = 14$ kg onto the piston, and release it. The piston and the weight on it start an oscillating motion with small amplitude, which can be regarded as harmonic. Determine the amplitude and the frequency of the oscillation, as well as the maximal speed of the piston.

(The wall of the cylinder can be considered a heat insulator. The external air pressure is $p_0 = 100$ kPa. If necessary, the approximation $(1 \pm x)^n \approx 1 \pm nx$ can be used, which is valid if x is close to zero, i.e., if $|x| \ll 1$.)

Problem 200. A gas, enclosed in a cylinder by a piston, is given a heat energy of $Q = 3988$ kJ, and as a consequence of this, the gas expands at constant pressure. The ratio of the specific heats measured at constant pressure and constant volume is $\gamma = 1.4$ for the gas. Determine how much of the absorbed heat increases the internal energy of the gas, and how much is given off in the form of work during the expansion.

2.3 First law of thermodynamics

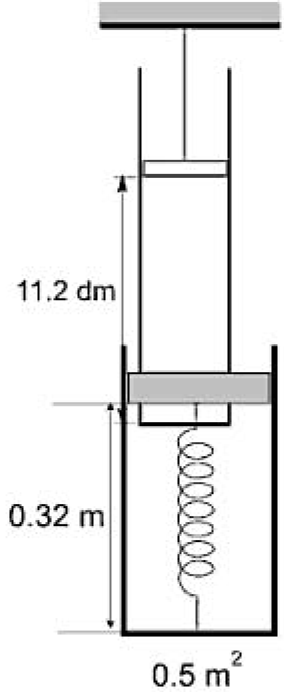


Problem 201. In a cylinder, whose cross-sectional area is $20\ \text{cm}^2$, a frictionless piston of mass 7.2 kg encloses a 33 cm high air column at 0 °C so that there is a 7 cm high empty part above the piston as shown. The atmospheric pressure is $10\ \text{N/cm}^2$, the densities of mercury and air in its initial state are $13.6\ \text{g/cm}^3$ and $1.8\ \text{g/dm}^3$ respectively, the specific heat of the air at constant volume is $0.7\ \text{J/(g K)}$. Use $g = 10\ \text{m/s}^2$.

a) Mercury is poured into the empty part above the piston until the cylinder is full. Find the mass of the mercury column. (Assume constant temperature.)

b) The air is then heated very slowly until all the mercury runs out from the cylinder. Find the minimum heat transferred to the air in this process.

Problem 202. A cylinder of mass 8 kg and cross-sectional area $20\ \mathrm{cm}^2$ is hanging, suspended on its piston. The cylinder contains helium of temperature $27\ ^\circ\mathrm{C}$. The temperature is slowly decreasing. How much heat is necessary to extract from the helium so that the initial length 11.2 dm of the gas column decreases to 8.96 dm? The external air pressure is 10^5 Pa, $g = 10\ \mathrm{m/s}^2$. The molar specific heat of helium at constant volume is $C_v = 12300\ \mathrm{J/(kmol \cdot K)}$.



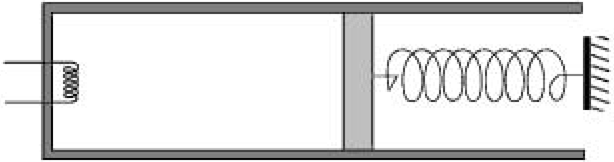
Problem 203. A cylindrical container of base area $0.5\ \mathrm{m}^2$ contains helium gas at 218.4 K. The gas is enclosed by a frictionless piston of mass 600 kg that is connected to the base of the container by a spring, whose spring constant is $2.67 \cdot 10^5\ \mathrm{N/m}$. Initially the piston is at a height of 0.32 m, which is the relaxed length of the spring. Atmospheric pressure is 10^5 Pa, the molar specific heat of helium is $C_v = 12.3\ \mathrm{joule/(mol\,K)}$, $g = 10\ \mathrm{m/s}^2$. The wall of the container is a good thermal conductor causing the temperature of the gas to change until it reaches the external temperature. The work done by the gas is found to be 1800 joules.

a) Find the external temperature.

b) Find the heat given to the helium gas.

Problem 204. In an 11.2 dm high cylindrical container, whose base area is 1 dm^2, a frictionless piston of mass 8 kg is held at a height of 5.6 dm. The piston encloses 1 mol of helium at 273 °C. The wall of the container is insulated. Find the maximum height reached by the piston after being released. The molar specific heat of helium at constant volume is $C_v = 12.6$ J/(molK), while at constant pressure it is $C_p = 21$ J/(molK). The atmospheric pressure is 10.12 N/cm^2.

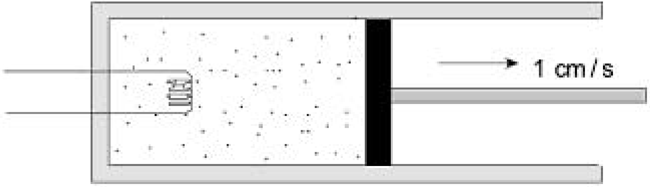
Problem 205. A piston encloses some air in the cylindrical vessel with horizontal longitudinal axis as shown in the drawing. The initial pressure of the air is equal to the external atmospheric pressure of 10^5 Pa. The cross-sectional area of the piston is 0.03 m^2. An originally unstretched spring with spring constant 2000 N/m is attached to the piston. The walls of the vessel and the piston are perfectly insulated. The initial volume of the enclosed air is 0.024 m^3, its initial temperature is 300 K. The air is heated to 360 K with a heating filament built into the vessel.



a) Find the displacement of the piston caused by the heating.

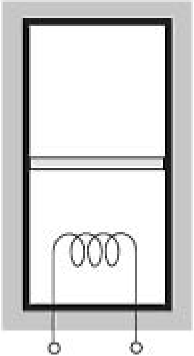
b) Find the energy delivered by the heating filament.

Problem 206. Ideal gas at pressure 10^5 Pa and volume 1 m^3 is enclosed by a piston in a cylinder. We start to move the piston outwards at a constant velocity of 1 cm/s. The cross-sectional area of the piston is 0.1 m^2. While the piston is moving, we can deliver heat to the gas through a heating filament.



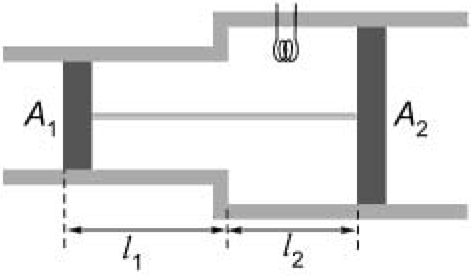
How should the heating power change as a function of time if we keep the temperature of the gas constant? (Apart from the heat transfer between the gas and the heating filament all other heat exchange can be neglected.)

Problem 207. Ideal gas which has degrees of freedom f, is part of a process that starts at T_0 and ends at $2T_0$ $V = aT^2$ (a = constant). Give the molar heat capacity as function of temperature.



Problem 208. A cylindrical container of volume 44.8 litres is divided into two equal parts by a horizontal frictionless piston. Each half of the cylinder contains 4 g of helium at 0 °C. The walls of the container and the piston are perfect insulators. There is a 220 V heater of resistance 242 Ω in the lower part. For how long should the heater be switched on to make the temperature of the helium in the upper part rise to 136.5 °C? The specific heats of helium are $c_p = 5230\ \mathrm{J/(kg \cdot K)}$ and $c_v = 3140\ \mathrm{J/(kg \cdot K)}$.

Problem 209. For a given amount of nitrogen gas the initial, minimum temperature is T_0, while the maximum temperature is $4T_0$. The gas is first heated at constant volume, then it is allowed to expand at constant pressure. Then it is cooled at constant volume and finally it is compressed at constant pressure. This way the gas returns to its initial state. Find the maximum possible efficiency of the cyclic process.



Problem 210. The walls of the two connecting cylinders shown in the figure are adiabatic (thermally insulating). The cross-sectional areas of the parts are $A_1 = 10\ \mathrm{dm}^2$ and $A_2 = 40\ \mathrm{dm}^2$. There is a well-fitting but freely moving, thermally insulating piston in each cylinder, at a distance $l_1 = l_2 = l = 1.5\ \mathrm{dm}$ from the point where the cross-sectional area changes. The pistons are fixed to each other by a thin and rigid rod. The enclosed volume contains air. The temperature and air pressure are $T_0 = 300\ \mathrm{K}$ and $p_0 = 10^5\ \mathrm{Pa}$ both inside and outside. The heater filament inside is operated for $t = 2$ minutes at a power of $P = 36\ \mathrm{W}$.

a) How much, and in what direction, will the pistons move until the new equilibrium position is reached?

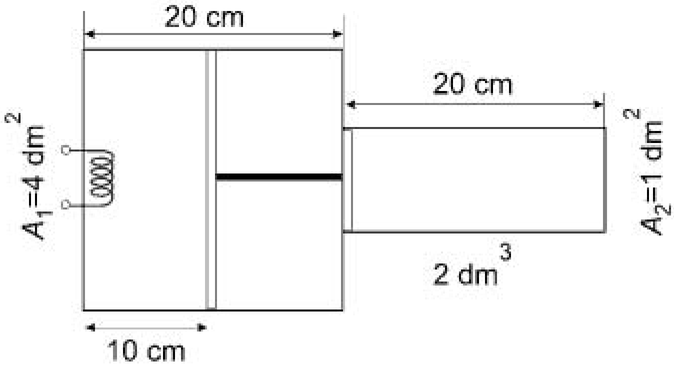
b) What will the temperature of the enclosed air be?

Problem 211. Consider the system of two pistons in two cylinders shown in the Figure. The cylinder walls and the pistons are good thermal insulators. Initially, the pressure of the air in all three compartments is $20\,\mathrm{N/cmn^2}$, and the temperature is $0\ ^\circ\mathrm{C}$. The filament in the leftmost compartment is heated for a short time. As a result, the pistons move $5\ \mathrm{cm}$ to the right.

a) What are the resulting pressures?

b) How much heat is given off by the filament?

The density of air at $0\ ^\circ\mathrm{C}$ and normal atmospheric pressure is $1.3\,\mathrm{g/dm^3}$, and its specific heat is $0.7\ \mathrm{J/(g\cdot K)}$ at constant volume and $0.98\,\mathrm{J/(g\cdot K)}$ at constant pressure.



Problem 212. In a closed container, there is a mixture of helium and oxygen gases of a total mass of 2.2 kg at a temperature of 0 °C. 143 500 joules of heat is added to the gas mixture. As a result, its temperature rises by 50 °C and its pressure increases by 13 740 pascals.

a) Find the mass of each gas.

b) Find the initial pressure of the mixture.

c) Find the volume of the container.

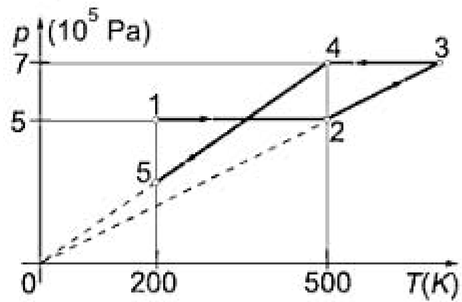
At constant volume, the molar specific heat of helium is 12 300 J/(kmol·K) and the molar specific heat of oxygen is 20 500 J/(kmol·K).

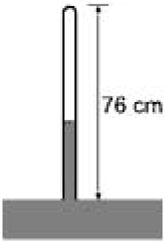
Problem 213. Hydrogen gas of mass $m = 20\,\mathrm{g}$ undergoes the processs 1–2–3–4–5 shown in the figure. The following data are given: $p_1 = p_2 = 5 \cdot 10^5$ Pa, $p_3 = p_4 = 7 \cdot 10^5$ Pa, $T_1 = T_5 = 200$ K, $T_2 = T_4 = 500$ K. (In the stages 2–3 and 4–5 of the process, pressure is directly proportional to temperature.)

a) Find the values of the volume in the states 1, 2, 3, 4, and 5, and the values of pressure and temperature not given.

b) Represent the process in both p-V and T-V diagrams.

c) Determine the net heat absorbed by the gas and the net work done on the gas during the whole process.





Problem 214. The upper end of a 76-cm-long glass tube is closed and the open lower end is submerged in mercury. The tube is partly filled with mercury, with 0.001 moles of air enclosed in the upper end. External atmospheric pressure can balance a mercury column of 76 cm. The molar specific heat of air is $C_v = 20.5\ \mathrm{J/(mol \cdot K)}$ at constant volume. How much heat is given off by the enclosed air while its temperature decreases by 10 °C?

Problem 215. A glass tube with thin walls is placed in a chamber of rarefied air. One end of the tube is closed and the other is covered by a stretched liquid film. The pressure of the air is p_0 and its temperature is T_0 both inside and outside the tube. The length of the tube is h, its radius is R. The surface tension of the liquid is α. The temperature in the tube starts to rise slowly.

a) At what temperature will the enclosed air have a maximum pressure?

b) How much heat is absorbed by the enclosed air until the state of maximum pressure is reached?

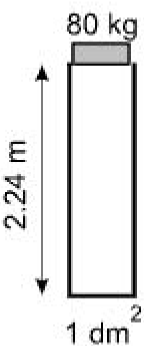
$R = 5\,\mathrm{mm}$, $h = 25\,\mathrm{mm}$, $T_0 = 250\,\mathrm{K}$, $p_0 = 1000\,\mathrm{Pa}$, $\alpha = 5 \cdot 10^{-2}\ \mathrm{J/m^2}$.
Assume that, in the pressure and temperature ranges investigated, the liquid is far away from its boiling point.

Problem 216. A piston of mass m encloses air with a pressure greater than the external atmospheric pressure in a horizontal cylinder whose walls are thermally insulated. If the piston is released, it can move in the cylinder without friction. In the adiabatic change that takes place, the maximum volume of the enclosed gas is twice as much as the original. Determine

a) the ratio of the minimum and maximum pressures of the gas,

b) the magnitude of the initial pressure.

(The pressure of the external air is $p_{ext} = 10^5$ Pa. The air can be considered as a gas with 5 degrees of freedom, therefore the ratio of its two specific heat capacities is $\gamma = c_p/c_v = 1.4$.)



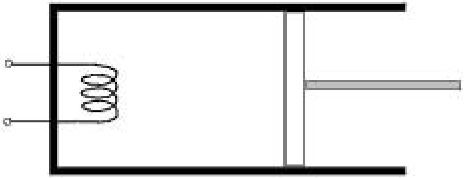
Problem 217. A 2.24 m high cylinder, whose base area is 1 dm^2 contains 4 g of helium gas at a temperature of 0 °C and pressure of 10 N/cm^2. An 80 kg piston is then dropped into the cylinder. Find the maximum speed of the piston if it moves without friction. There is no heat transfer between the gas, the cylinder and the piston because of the rapidity of the process. Use $g = 10\,m/s^2$. The specific heats of helium are: $c_v = 3150$ J/(kg·K), $c_p = 5250$ J/(kg·K).

Problem 218. Initially $n = 10$ mol of an ideal gas has the pressure $p_1 = 10^5$ Pa, volume $V_1 = 249.42 \text{ dm}^3$ and temperature $T_1 = 300$ K. Then the gas is heated, and in an isobaric process it reaches the temperature T_2. During this process the work done by the gas is 68% of the increase of its internal energy. If, however, from the same initial state an adiabatic compression is used to increase the temperature of the gas to T_2, then $W = 36.85$ kJ has to be done on the gas.

a) What type of gas is the experiment performed with?

b) Determine the final temperature T_2.

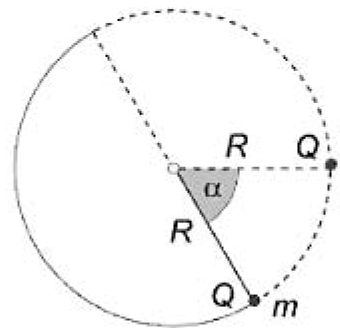
Problem 219. There is 5 g of a certain diatomic gas in a container closed with a frictionless piston. The gas is heated for 25 seconds by an electric resistor of 50 Ω built in the container, applying a voltage of 220 V. While the gas expands at constant pressure, its temperature increases by 250 °C. The efficiency of the electric heater is 75%. What kind of gas can be found in the container?



Problem 220. A container, closed by a freely moving piston, contains a mixture of hydrogene and helium gas of total mass $m = 180$ g. A heat of $Q = 156$ kJ is transferred to the gas at constant pressure. Due to this the gas performs 56 kJ work. Determine the mass of hydrogene in the mixture. Determine the temperature change of the system.

Problem 221. The state of helium gas is changed in such a way that its graph is a straight line segment on the pressure–volume plane. During this process the total heat transferred to the gas is equal to the heat necessary to double the absolute temperature of the gas at constant volume. By what ratio may the volume of the gas most increase? (The expression "total heat" refers to the signed sum of heat absorbed and heat released during the process.)

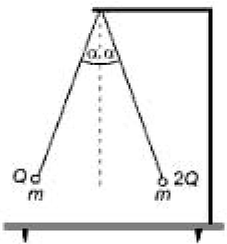
Problem 222. A small ball of mass $m = 1$ g and charge Q is attached to the end of a string of length $R = 10$ cm. Level with the suspension point of the pendulum, at a distance of $R = 10$ cm there is a small fixed object of the same charge Q. If the pendulum is released from a position $\alpha = 60°$ below the horizontal, the string will become slack when the pendulum bob has covered a semicircle exactly. Find the magnitude of the charge Q.



Chapter 3

Electrodynamics Problems

3.1 Electrostatics



Problem 223. Two small metal balls of mass $m = 0.1$ g are suspended at the same point by insulating threads of length $l = 30$ cm. One of the balls is loaded with twice as much electric charge as the other. Pushing the balls towards each other by insulating materials, we move them to a position where both threads make an angle of $\alpha = 20^\circ$ with the vertical, and the threads remain in a common vertical plane. After releasing the two balls from this position at the same time, the angle between the two threads reaches the largest value of $\beta = 84^\circ$.

Determine the charge of the balls.

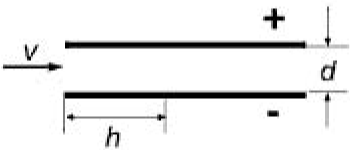
Problem 224. In free space, far from all celestial objects, two particles, one of mass $m_1 = 6 \cdot 10^{-12}$ kg and of charge $Q_1 = 2.43 \cdot 10^{-13}$ C and the other of mass $m_2 = 1.2 \cdot 10^{-11}$ kg and of charge $Q_2 = -2.43 \cdot 10^{-13}$ C move at a constant speed in such a way that the distance $d = 1.5$ cm between them is also constant. How is this possible? Determine the speed of the particles.

Problem 225. In free space two specks of dust, one of mass $m_1 = 1.7 \cdot 10^{-11}$ kg and of electric charge $Q_1 = 10^{-9}$ C and the other of mass $m_2 = 1.3 \cdot 10^{-11}$ kg and charge $Q_2 = -5 \cdot 10^{-9}$ C are released at a distance $d_1 = 6$ cm from each other with zero initial speed.

a) Where will the two specks of dust meet?

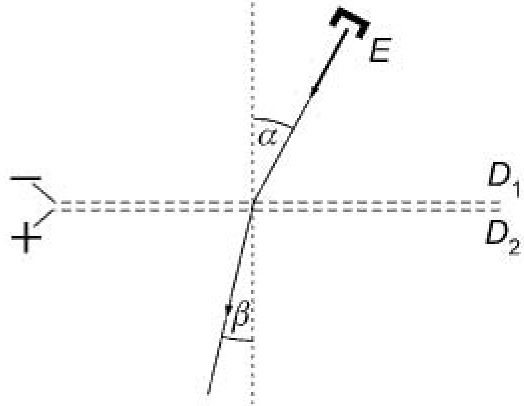
b) Determine the speed at which the specks approach each other, when they are at the distance $d_2 = 1$ cm.

Problem 226. A capacitor has plates of large area separated by $d = 3$ cm.



The potential difference between the plates is $V = 60000$ V. What should be the speed of a small object of charge $Q = 4 \cdot 10^{-3}$ C and mass $m = 5 \cdot 10^{-6}$ kg shot horizontally into the uniform field at the height of half the plate separation d, so that it reaches one of the plates at a distance of $h = 12$ cm?

Problem 227. A potential difference V is applied between two finely woven parallel wire meshes D_1-D_2 with the polarity shown in the figure. Electrons originating from electron source E arrive at mesh D_1 at velocity v.

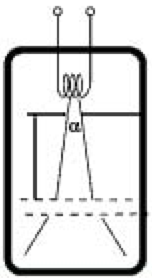


a) Show that for the angle of incidence α and the angle of refraction β Snell's law is valid (ratio $\sin\alpha/\sin\beta$ is independent of the angle of incidence and has the same value for every electron).

b) Determine the value of the refractive index.

(You can assume that the electric field between the meshes is homogenous and everywhere else the electric field is zero. $v=3\cdot 10^6$ m/s, $V=25$ V.)

Problem 228. In a vacuum tube electrons accelerated through a potential difference V_0 leave the anode with beam angle α. A metal lattice pair is then placed in the way of the beam. What potential difference should be applied to the lattices if we want the electron beam to form an angle of 2α after leaving the lattice-pair? Values: $V_0=60000\,\text{V}$, $\alpha=30°$.



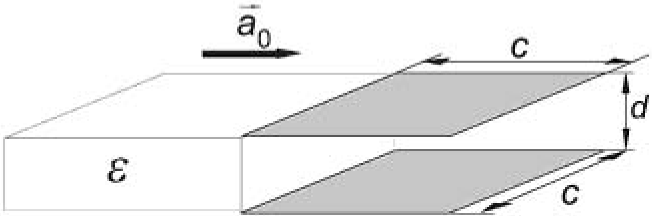
Problem 229. A particle of charge $Q = +10^{-5}$ C is fixed. A second particle of mass $m = 0.01$ g and charge $q = +10^{-7}$ C standing at infinity is given a velocity of $v_0 = 200$ m/s in a direction whose line passes at a distance of $d = 0.1$ m from the fixed charge.

a) Find the smallest separation between the two charges.

b) What should the value of d be if the final velocity of the moving charge is now perpendicular to its initial velocity v_0?

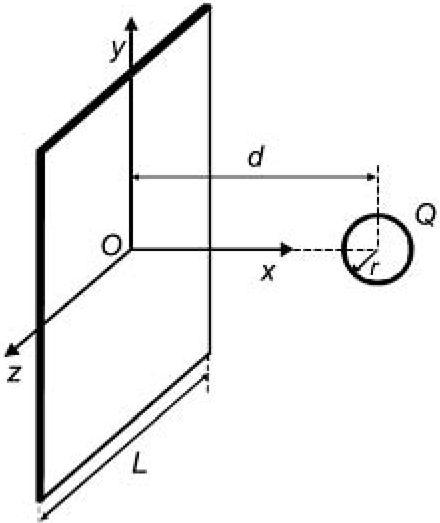
Problem 230. If the cathode of a photocell is illuminated with a light of increasing frequency, the anode current will start at a frequency of $3 \cdot 10^{14}\ \frac{1}{s}$. A capacitor with capacitance 1 pF is connected between the anode and the cathode of this photocell, and the cathode is illuminated with light of wavelength 425 nm. Assuming that the illumination is long enough, find the number of electrons arriving on the anode.

Problem 231. Two metal spheres of equal mass and radius are suspended from a common point, with two insulating threads of equal length. If the spheres are loaded with equal electric charges, and submerged into paraffin, the angle between the two threads is $2\alpha_P = 60°$. If the paraffin bath is removed, i.e., the charged spheres are in the air, then the angle between the threads is $2\alpha_A = 70°$. Determine the density of the spheres. (Paraffin is an electric insulator, its relative permittivity is $\varepsilon_r = 2$, and its density is $\varrho_p = 800\ \mathrm{kg/m^3}$. The diameter of the spheres is much less than the length of the threads.)



Problem 232. A parallel plate capacitor consists of a pair of square plates with sides c that are at distance d apart. The capacitor is connected to a generator of constant voltage U. The space between the plates is originally filled with air. A plate with relative dielectric constant ε_r is inserted between the plates at a constant acceleration a_0 as shown in the figure. The insulator starts from the edge of the plate, from stationary position. Determine the charge-time function of the capacitor and the charging current-time function. Sketch the shape of the functions. Calculate the maximum value of the charge on the capacitor and of the charging current.

($c = 20\,\mathrm{cm}$, $d = 2\,\mathrm{mm}$, $U = 100\,\mathrm{V}$, $a_0 = 2\,\mathrm{m/s^2}$, $\varepsilon_r = 101$.)



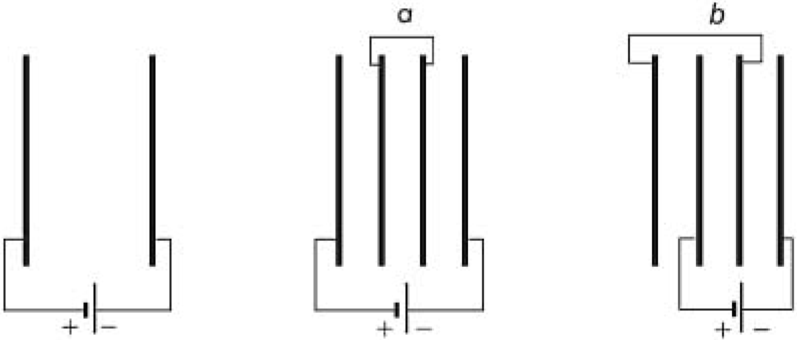
Problem 233. A big insulating square plate of size L and negligible width is uniformly charged with a charge of 100 Q. Let the plane of the plate be the $y-z$ coordinate plane. There is a hollow insulating sphere of radius r with centre at the point $(d,0,0)$ in front of the plate. The sphere has a thin wall, and is uniformly charged with charge Q. Determine the electric field at the interior points of the sphere, and at the point $(d/2,d/2,0)$, provided that $L=100d$ and $r=d/5$. Express these results in terms of Q, d and the dielectric constant ε_0.

Problem 234. Two identical air capacitors with capacitance C are connected in series to a battery with constant voltage V. Find the change in the energy of the capacitors, of the battery and of the surroundings

a) if the distance between the plates of one of the capacitors is increased to twice the original distance using an insulating handle,

b) an insulator with dielectric constant $\varepsilon=2\varepsilon_0$ is inserted between the plates of one of the capacitors.

Problem 235. A parallel-plate capacitor is connected to a battery which builds up an electric field of 600 V/m between the plates.



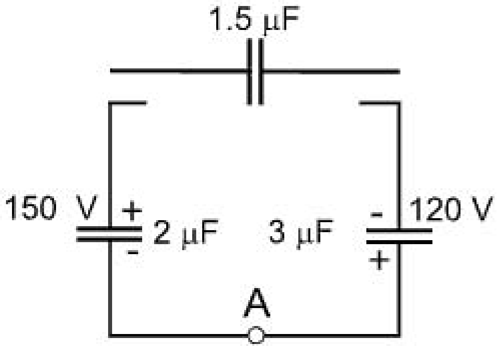
a) In the first part, two plates are placed into the capacitor as shown. The plates, that are initially neutral, are connected by a wire and are positioned such that the four plates are at equal distance from each other. Find the electric field strengths between the plates.

b) In the second part, the two initially neutral plates that are connected are positioned as shown in figure b). The plates are at equal distances from each other in this case as well. Find the electric field strengths between the plates.

Problem 236. One plate of a 2-μF capacitor charged to 150 volts is connected to the oppositely charged plate of a 3-μF capacitor charged to 120 volts. The other plate of each capacitor ends in a free wire. An uncharged capacitor of 1.5-μF is dropped onto the free ends.

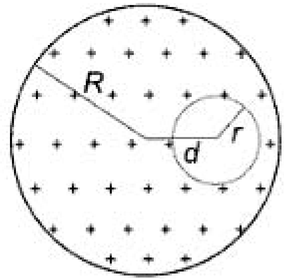
a) What will be the potential difference across each capacitor?

b) How much charge will pass through point A and in what direction?

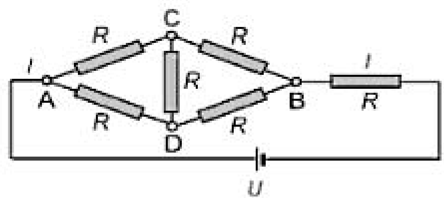


Problem 237. In ancient times, people believed that the Earth was a big, flat disc. Let us imagine that the Earth is not actually sphere with radius R but a flat disc with a very large radius and a thickness of H. What thickness H is needed to experience the same gravitational acceleration on the surface of the disc (far from its rim) as on the surface of the spherical Earth? ($R = 6370\,\text{km}$. Let us consider the densities in the two 'Earth' models to be constant and equal to each other.)

Problem 238. A long insulating cylinder of radius R has a cylindrical bore of radius r in it. The axes of the cylinder and the bore are parallel, separated by a distance d. The insulator carries a positive charge of uniform distribution with a charge density of ϱ. The relative dielectric constant of the material is 1. Find the electric field inside the bore.

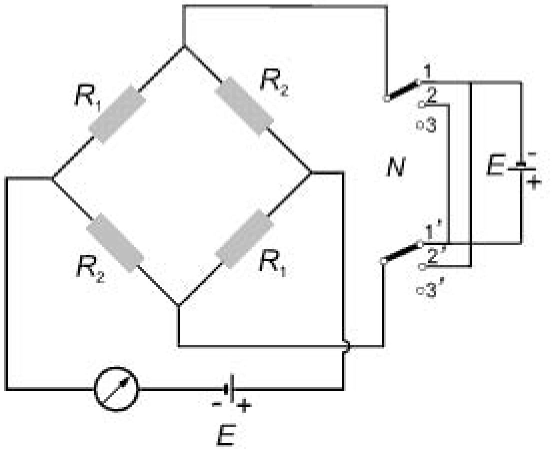


Problem 239. If the values of the resistance of the resistors shown in the figure are equal then the current in the main branch is I. By what factor does this current change if the resistance of the two resistors, which are diagonally opposite each other, is doubled?

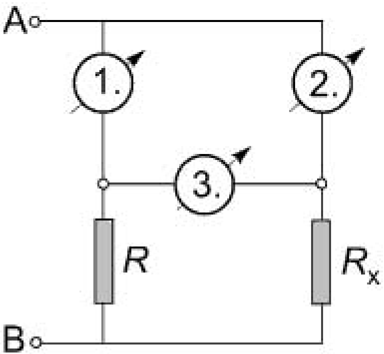


3.2 Direct current

Problem 240. We have two rechargeable batteries available, their electromotive forces and their internal resistances are the following: $U_{01} = 12.6$ v, $R_1 = 0.05$ ohm, $U_{02} = 12.2$ V, $R_2 = 5$ ohm. What will happen if the two rechargeable batteries are connected in parallel and the circuit is closed through a resistor of resistance $R_k =$ $= 2$ ohm?



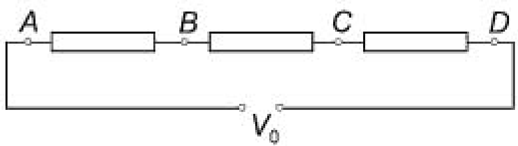
Problem 241. The figure shows an electric circuit which contains a double switch labelled by S. When the switch is in the position $11'$ the ammeter reads $I_I = 6$ A, and when the switch is in position $22'$ the reading is $I_{II} = 3$ A. What is the reading on the ammeter when the switch is in position $33'$? The electromotive forces E of the two batteries are equal, their internal resistances and the internal resistance of the ammeter are negligible.



Problem 242. In the circuit shown the three ammeters are identical, each have a resistance $R_0 = 2\,\Omega$. Between points A and B there is a constant potential difference of $19\ \mathrm{V}$. The first and second ammeter read $I_1 = 2.5\,\mathrm{A}$ -t and $I_2 = 1.5\,\mathrm{A}$ respectively.

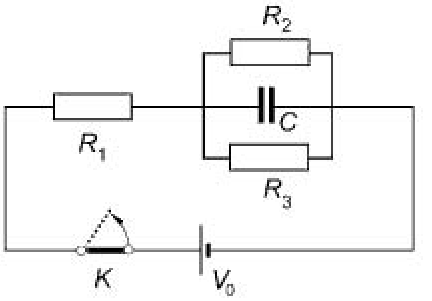
a) What does the third ammeter read?

b) Investigate what happens to current I_3 if the value of R_x is changed.



Problem 243. Three resistors are connected in series with a battery of internal voltage $V_0 = 62\ \mathrm{V}$ and internal resistance $R_b \approx 0$ as shown in the figure, then measurements are carried out with a single voltmeter. The results of the first three measurements are: $V_{AB} = V_{BC} = V_{CD} = 20\ \mathrm{V}$. What does the instrument show when it is connected between point pairs AC and AD?

Problem 244. In the circuit shown in the figure switch K is kept closed for a long time and then it is opened. The constant voltage across the terminals of the battery is $V_0 = 9$ V, the capacitance of the capacitor is $C = 50\ \mu$F, the values of resistances R_1 and R_2 are equal, $R_1 = R_2 = 100\ \Omega$.



a) Find the charge that flows through resistor R_3 after the switch is opened if the value of resistance R_3 is $400\ \Omega$.

b) Find the maximum charge that can flow through resistor R_3 after the switch is opened, if the value of resistance R_3 is chosen suitably.

Problem 245. We have N identical cells whose no-load voltage is V_0 and internal resistance is R. We create a battery from these in the following way: first we connect a given number of cells with the same polarity in series into a chain, and then connect these chains containing the same number of cells in parallel with the same polarity. Then we connect a consumer whose resistance is optimal for maximum power output to the acquired battery.

a) Find the number of batteries that should be connected into a chain, that is, find the arrangement of the cells in order to acquire the maximum power output on the consumer.

b) Find this power if $N = 64$, $V_0 = 12$ V, $R = 2\ \Omega$.

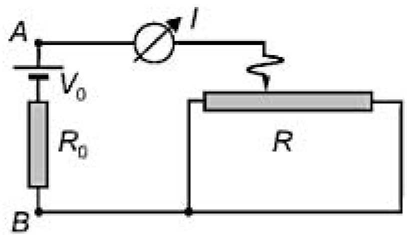
Problem 246. A $100-\Omega$, $2-\mathrm{W}$ resistor is to be operated from a variable voltage supply. A variable resistor of resistance $1000\,\Omega$, which has three terminals, and which can be loaded by $15\,\mathrm{W}$, and a voltage supply of $48\,\mathrm{V}$, whose internal resistance is negligible, are given. In what interval can the voltage across the resistor be varied?

Problem 247. The range of voltage of a meter used as a voltmeter can be changed to n times its original value with the help of a $27\,\Omega$ multiplier. Using the same meter as an ammeter, its range of current can be changed to n times its original value using a $3\,\Omega$ shunt.

a) Find the internal resistance of the meter.

b) The power dissipated by the moving-coil of the meter when giving a full-scale reading is $9\cdot 10^{-4}$ W. Find the voltage and current across the moving-coil.

Problem 248. The circuit diagram shows an emf source with an electromotive force V_0 and an internal resistance R_0 between points A and B. The total resistance of the variable resistor is R, which is greater than $4/3$ times the internal resistance of the emf source. The resistance of the ammeter and the other wires can be neglected. If the sliding contact is moved along the variable resistor, the ammeter shows a changing current.

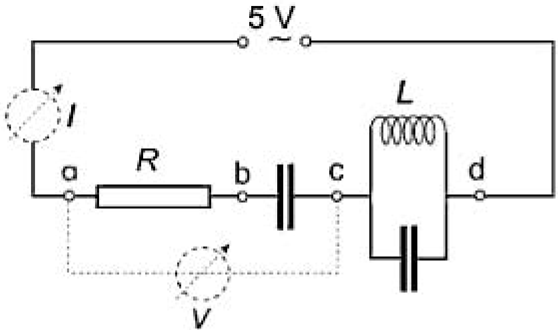


Show that the minimum current that can be measured is smaller then $3/4$ times the maximum current.

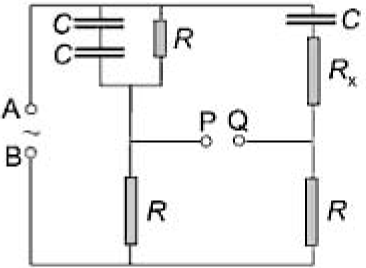
Problem 280. Alternating voltages of various angular frequencies are connected between the two terminals of a closed box. The impedances are measured and tabulated below:

ω [s^{-1}]	20	200	250	300	325	350	400	1000	5000
Z [Ω]	782	53.2	34.0	25.4	25.2	27.2	34.9	145.5	792

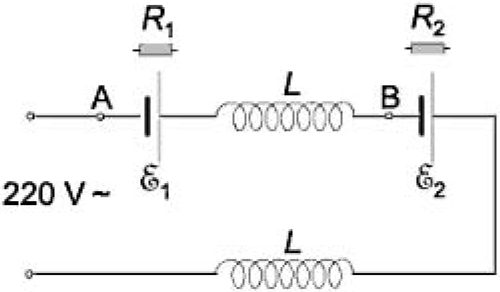
What does the box contain?



Problem 281. The circuit shown in the figure is connected to an AC generator that supplies 5 V. $R = 5\ \text{k}\Omega$ and the two capacitors are identical. The ammeter reads 1 mA while the voltmeter reads 13 V. What will the meters read if the angular frequency of the generator is changed from ω to $\omega/\sqrt{2}$? Assume the meters to be ideal. (The voltmeter's impedance is infinitely large, the impedance of the ammeter is negligible.)



Problem 282. In the circuit shown $C = 2\mu\text{F}$, $R = 1\ \text{k}\Omega$ and R_x is a resistor of unknown resistance. An alternating potential is applied across AB. Under what conditions will we not hear any sign of a potential difference in a sensitive headphone connected across points PQ?

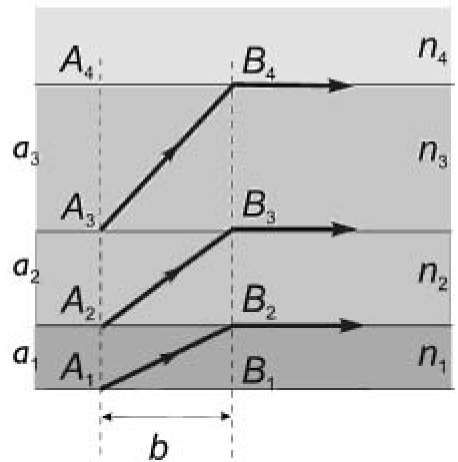


Problem 283. Two coils of equal inductance and two gavlanic cells of emfs $\mathcal{E}_1 = 50\ \text{V}$ and $\mathcal{E}_2 = 100\ \text{V}$ are connected to a 220 V AC outlet as shown. The internal resistances of the cells are not negligible. In the circuit the current lags the potential difference by 45°. A DC voltmeter connected across points A and B reads zero. What will an AC voltmeter read when connected across points A and B?

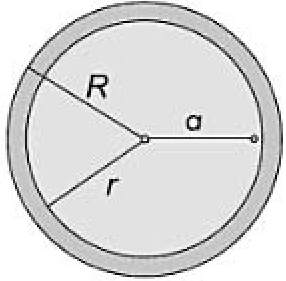
Optics Problems

Problem 284. Two power supplies with the same output voltage U are connected in a series. We then gain a power supply whose output voltage is also U. Can it happen?

Problem 285. Four layers of glass plates are placed on top of each other in such a way that the bottom one has thickness a_1 and refractive index $n_1 = 2.7$, the next one has thickness a_2 and refractive index $n_2 = 2.43$, and the third one and the top one have thicknesses a_3 and a_4 and refractive indices n_3 and n_4 respectively. Three rays of light starting simultaneously from points A_1, A_2, A_3 reach points B_2, B_3, B_4 at the same time, with their angles of incidence being the critical angles as shown. $A_2B_2 = A_3B_3 = A_4B_4 = b = 10$ mm. Find thicknesses a_1, a_2, a_3 and refractive indices n_3, n_4.



Problem 286. A glass spherical shell with an outer radius of $R = 7.5$ cm and an inner radius of $r = 6.5$ cm has a refractive index $n_2 = 1.5$. The inside of the shell is filled with carbon disulphide, whose refractive index is $n_1 = 1.6$. A source of light is placed at a distance of $a = 6$ cm from the centre. What percent of the energy of the light source leaves the system?



Problem 287. The optical model of an endoscope is an optical fibre of refractive index n_1, which is covered by a cladding of refractive index n_2. The end of the fibre is flat and it is in contact with the surrounding material of refractive index n_3. (The refractive indices are with respect to air.) How should the value of n_1 be chosen if through the fibre the whole half-space below the end of the fibre is to be visible

a) $n_2 = n_3 = 1$,
b) $n_2 = 1$ and $n_3 = 4/3$?

Problem 288. We have three equal lenses of focal length f. By placing these lenses at distances d_a and d_b from each other we build an optical system. With this optical system the image of an object is detected on a screen, which is at distance A from the object. We observe that when moving the optical system along the optical axis back and forth the image on the screen remains sharp. By what values of the geometric data is this possible?

Problem 289. If we accommodate our eye to infinity and look into a telescope the image of the Sun would be clear. If a sharp image of the sun is to be created on a screen which is 16 cm from the telescope, how far a distance must the image of the telescope be moved? The absolute value of the focal length of the eyepiece is 2 cm.

Problem 290. You have three converging lenses. Their focal lengths are 90 cm, 10 cm and 8 cm. How can you build a telescope from them, with the with the greatest magnification, if the maximum length of the telescope is 150 cm? (The lenses are all thin lenses and lens aberrations can be neglected.)

Problem 291. At one end of a 50 cm long tube there is a converging lens of optical power 2 dioptres and at the other end there is a diverging lens of optical power −2 dioptres. A plane mirror is placed behind the diverging lens at a distance of x, perpendicularly to the axis of the tube. For which distance of x can the real image of object, which is placed in front of the converging lens at a distance of 100 cm, be in the plane of the object? What is the magnification, and is the image inverted or erect?

Problem 292. There is a hole in the middle of small thin circular converging lens of focal length $f = 4$ cm. The diameter of the hole is half of the diameter of the lens. There is a pointlike light-source $A = 9$ cm away from a wall. Where should the lens be placed in order to get a single, circular illuminated spot on the wall, which also has a sharp edge?

Problem 293. A 20 cm long light tube lies on the principal axis of a converging lens with diameter $2R = 4$ cm and focal length 40 cm. The ends of the tube are at distances 60 cm and 80 cm from the lens. Where should a screen (which is perpendicular to the principal axis) be put on the other side of the lens, if the diameter of the light spot on it is to take its minimum value? Find the minimum diameter of the light spot.

Problem 294. The following objects are placed after each other onto a central axis with a separation of 4 dm each, a point source of light (O), a diverging lens of focal length -4 dm, a converging lens of focal length $+4$ dm and a concave mirror of focal length 8 dm. The diameter of the lenses and mirror is $d = 2$ dm. The point source of light is then moved perpendicular to the central axis. What should its perpendicular displacement (x) be if the image is to be captured on a screen?

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